Wide Field Radio Interferometry for Cosmology

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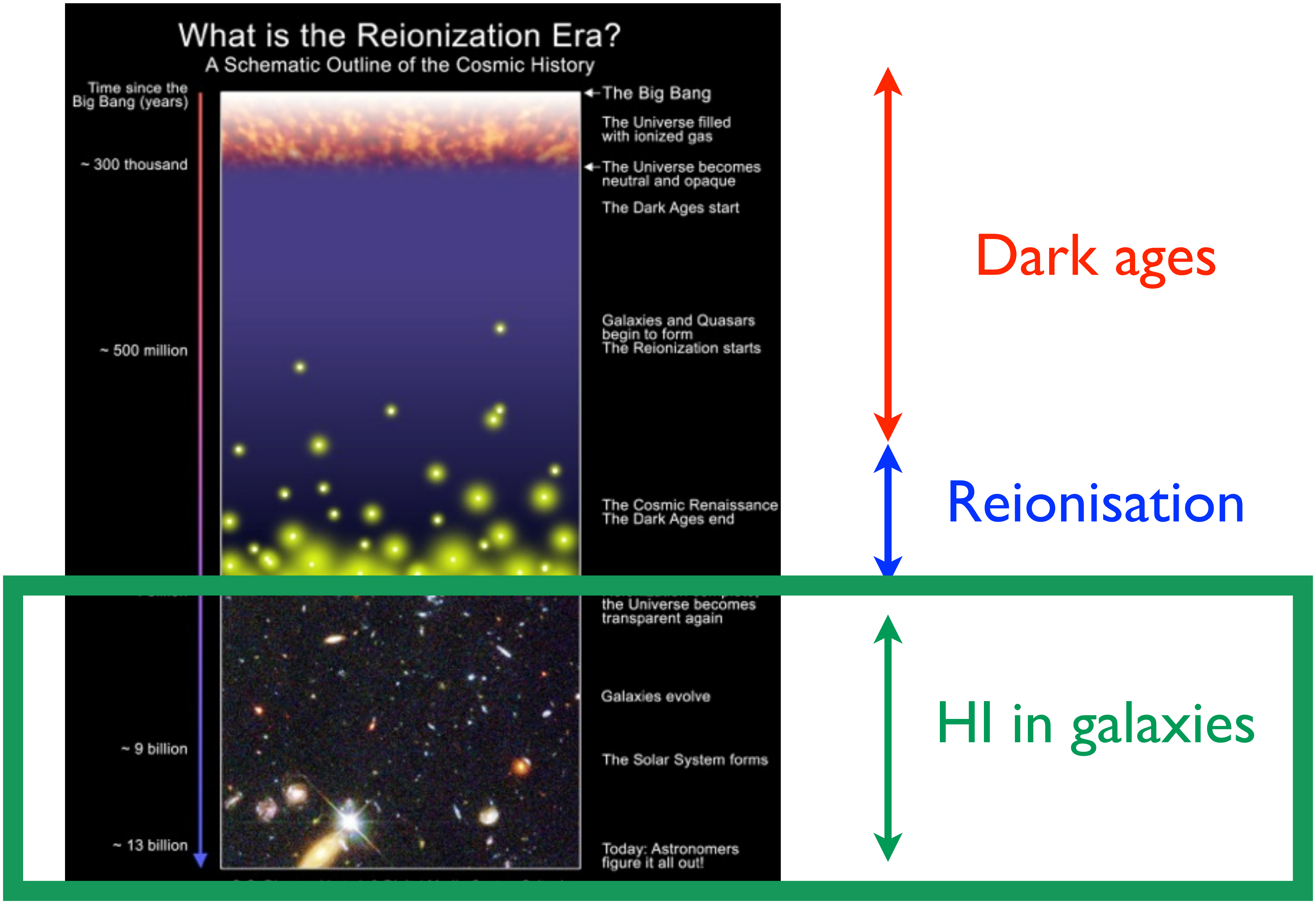
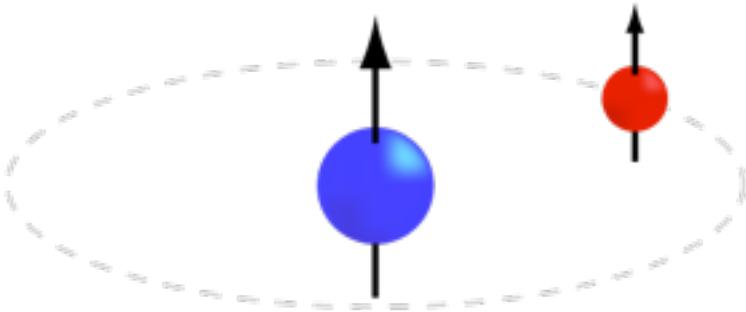
History of Hydrogen



Cosmological 21cm



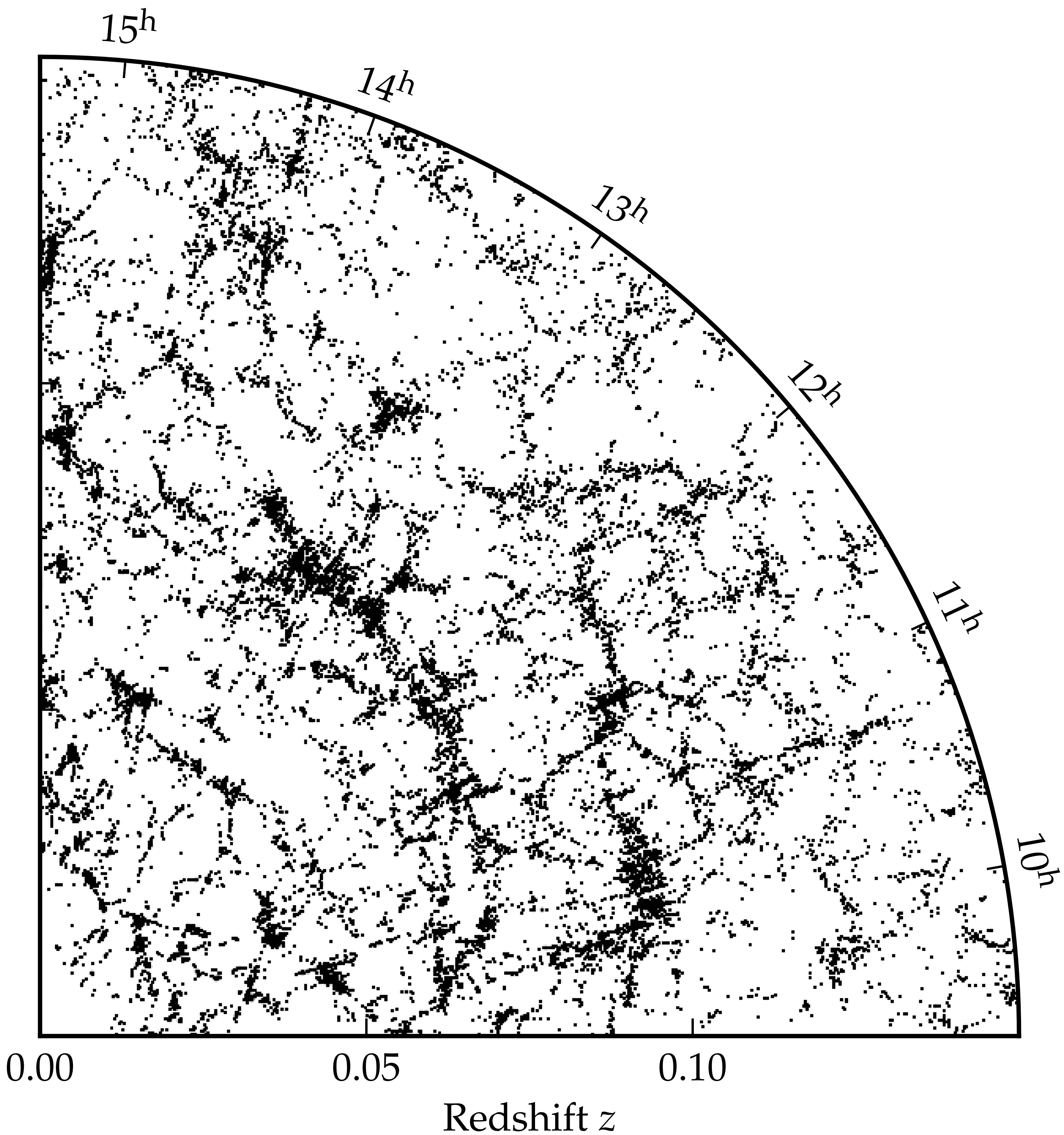
- 21cm line is the transition between parallel and anti-parallel spins in the H ground state
- The ratio between the two occupancies determines the spin temperature T_S

$$n_1/n_0 = (g_1/g_0)\exp(-T_*/T_S)$$

- We can observe the contrast relative to the CMB

$$\Delta T = 23.8\left(\frac{1+z}{10}\right)^{1/2}\left[1-\bar{x}(1+\delta_x)\right](1+\delta_b)(1-\delta_v)\left[\frac{T_S-T_\gamma}{T_S}\right]\ \mathrm{mK}$$

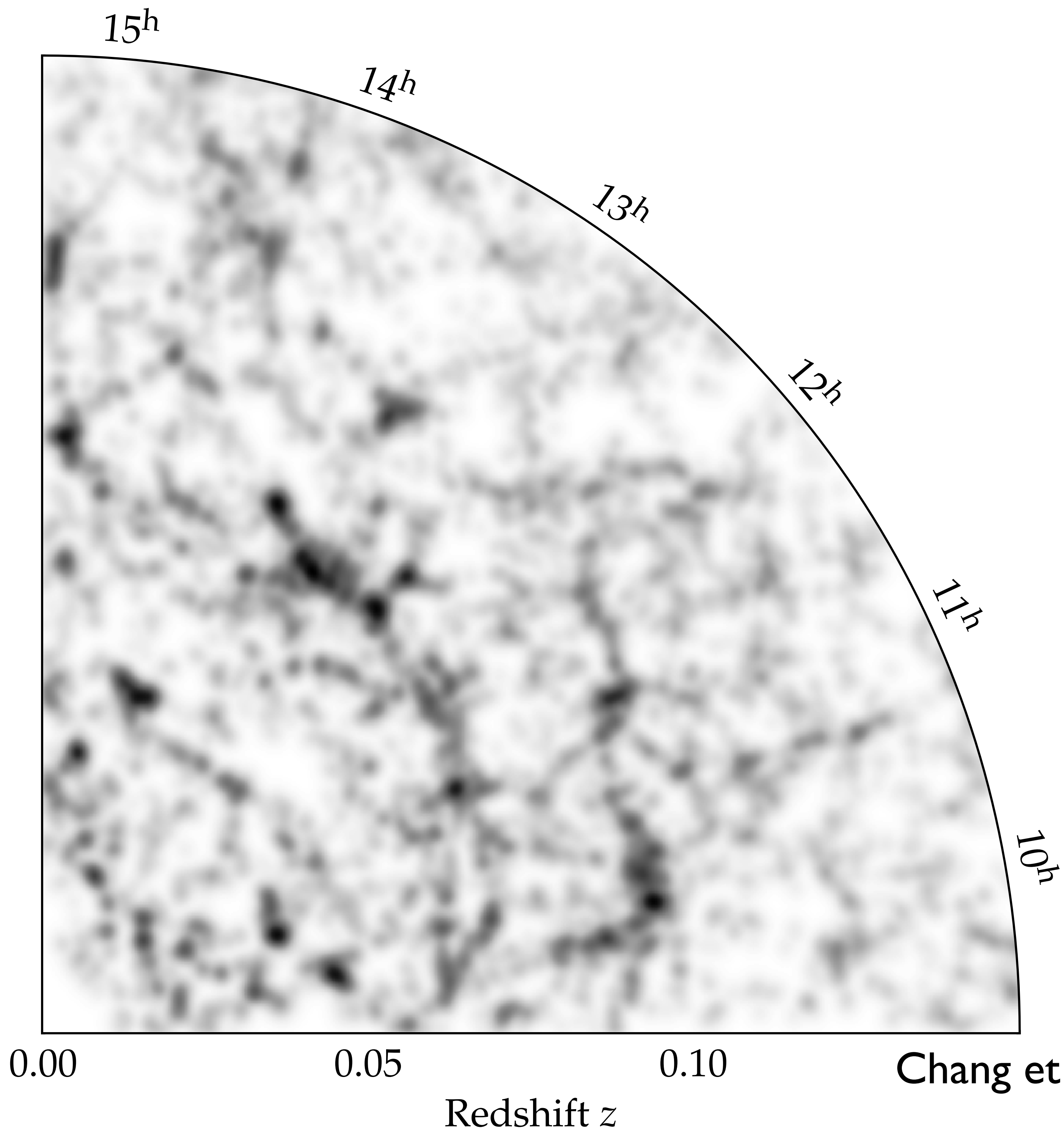
Traditional survey



- Detect all galaxies with high significance.
- Take spectra to determine redshift

Only interested in large scales

Intensity Mapping



- Observe galaxies with a line transition
- Automatically gives redshift

Don't need to resolve individual galaxies

Chang et al, 2008; Wyithe and Loeb 2008

21cm Intensity Mapping

- In 21cm the frequency gives the redshift.
- Observe the diffuse emission from many unresolved galaxies
- Changes the game in telescope design:
  - Previously: large field of view, large collecting area, large angular resolution (SKA?)
  - Now: large field of view, large collecting area, modest angular resolution (compact arrays, single dishes).

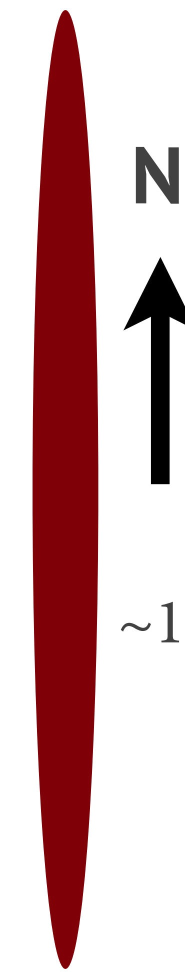
Chang, Pen, Peterson and McDonald , 2008, http://arxiv.org/pdf/0709.3672

Traditional Interferometer

Small field of view
High resolution imaging

Survey Interferometers

chime



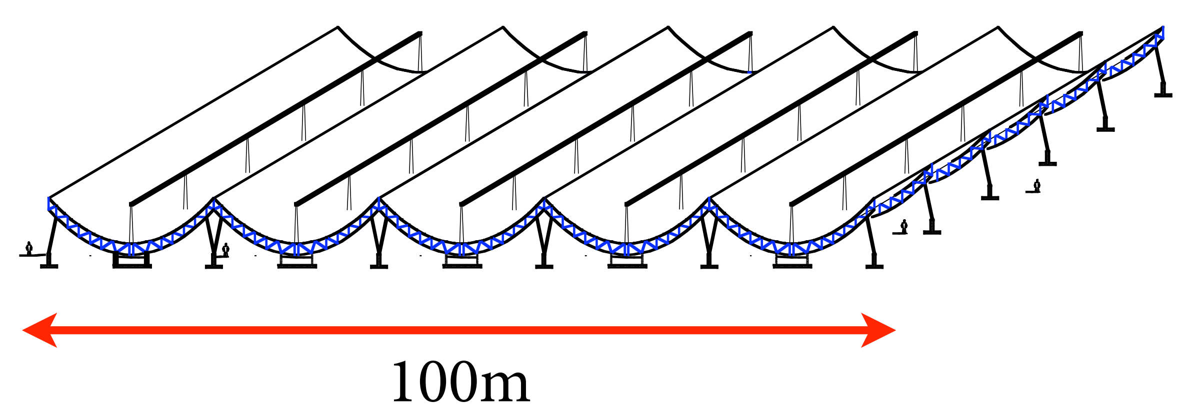
Beam:
~120 x 2 deg

Large field of view
Low resolution surveys

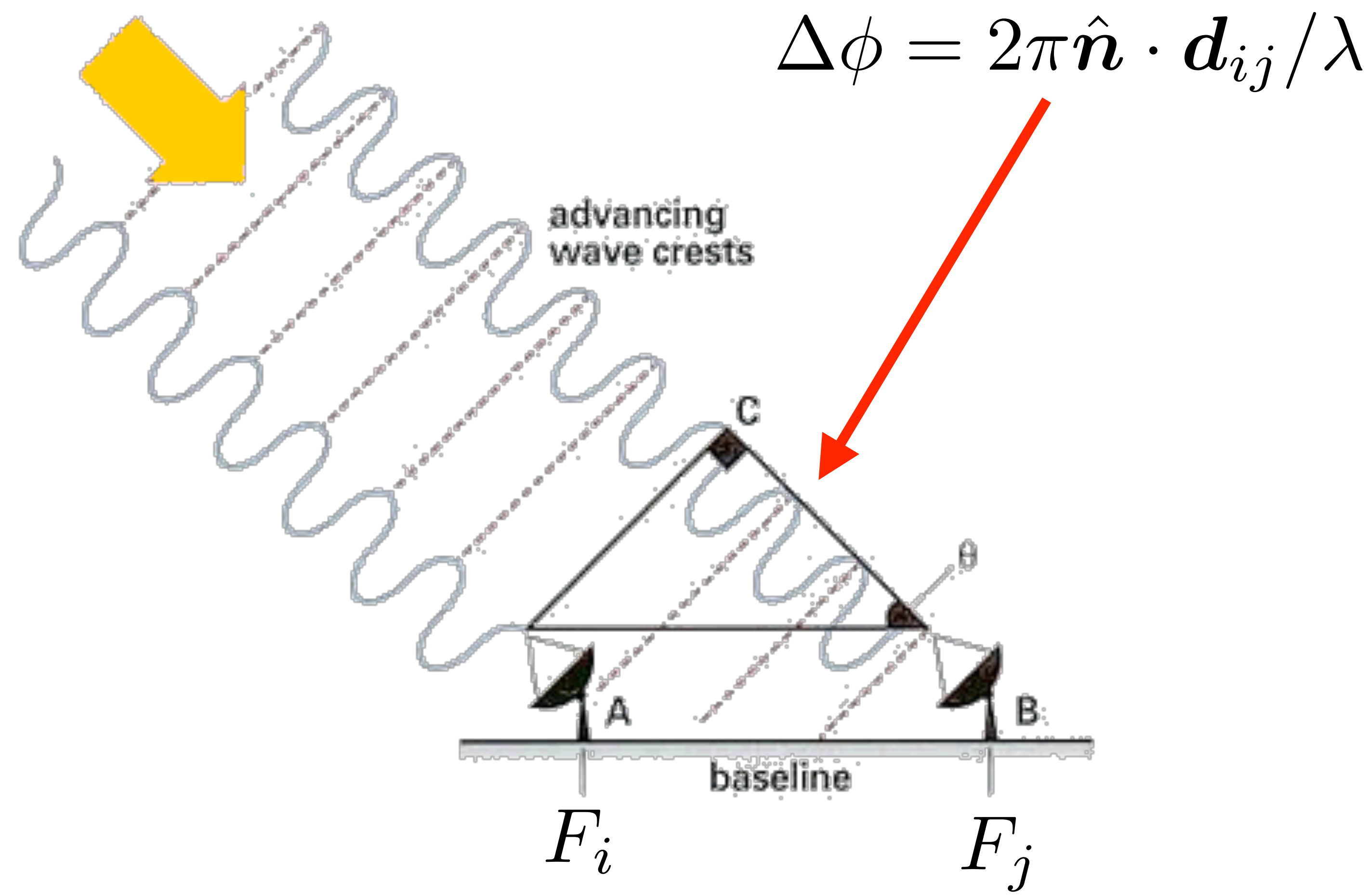
21cm Drift Scan Interferometers

- Transit instrument, no moving parts, time variation comes only from Earth rotation, plus noise.
- Most experiments have large challenges:
  - Wide field at given instant *(~ 180 x 1 degrees)*
  - All sky as total surveyed area is large *(3π sr)*
  - Data volume *(many TB/day)*
  - Polarisation leakage
  - Foreground removal

CHIME



Interferometers



- Visibility is instantaneous correlation of 2 antennas

$$V_{ij} = \langle F_i F_j^* \rangle$$

- Written explicitly:

$$V_{ij}(t) = \frac{1}{\Omega_{ij}} \int d^2\hat{\mathbf{n}}\, A_i(\hat{\mathbf{n}}; t) A_j^*(\hat{\mathbf{n}}; t) e^{2\pi i \hat{\mathbf{n}} \cdot \mathbf{u}_{ij}(t)} T(\hat{\mathbf{n}})$$

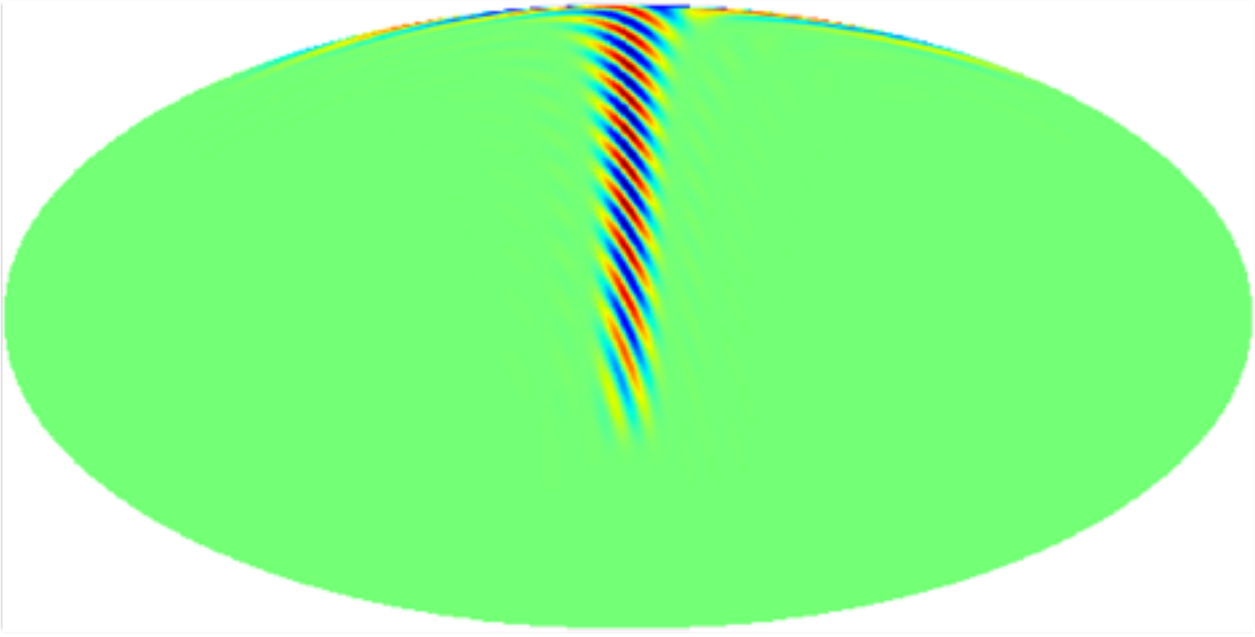
- Traditional analysis approximates this to a 2D Fourier transform and proceeds from there.

Interferometers

- Write in terms of a beam transfer function:

$$V_{ij}(t) = \frac{1}{\Omega_{ij}} \int d^2\hat{\mathbf{n}}\, A_i(\hat{\mathbf{n}};t) A_j^*(\hat{\mathbf{n}};t) e^{2\pi i \hat{\mathbf{n}}\cdot\mathbf{u}_{ij}(t)} T(\hat{\mathbf{n}})$$

$$V_{ij}(t) = \int d^2\hat{\mathbf{n}}\, B_{ij}(\hat{\mathbf{n}};t) T(\hat{\mathbf{n}}) + n_{ij}(t)$$



Transit Interferometers

- Timeseries is periodic on the sidereal day $t \to \phi$
  - Apply this restriction and see how the analysis goes.

$$V_{ij}(\phi) = \int d^2\hat{\mathbf{n}}\, B_{ij}(\hat{\mathbf{n}};\phi) T(\hat{\mathbf{n}}) + n_{ij}(\phi)$$

Spherical Harmonic Transform

$$V^{ij}(\phi) = \sum_{lm} B^{ij}_{lm}(\phi) a^T_{lm} + n^{ij}(\phi)$$

Fourier Transform

$$V^{ij}_m = \sum_{l} B^{ij}_{lm} a^T_{lm} + n^{ij}_m$$

m-mode transform

- Mapping does not mix m's (each is independent)

$$V_m^\alpha = \sum_l B_{lm}^\alpha a_{lm}^T + n_m^\alpha$$

- Write in vector form

$$\mathbf{v} = \mathbf{B}\,\mathbf{a} + \mathbf{n}\ .$$

- Simple, linear mapping from the information on the sky, to the measured degrees of freedom
- Discrete relation, with finite number of degrees, can apply all the standard statistical, signal processing techniques to it.
- Computationally efficient: For 1000 m's an $O(N^3)$ matrix operation becomes 10^6 times faster

Polarisation

- Extends easily to polarisation. Measurement equation:

$$V_{ij}(\phi) = \int \Big[B^T_{ij}(\hat{\mathbf{n}};\phi) T(\hat{\mathbf{n}}) + B^Q_{ij}(\hat{\mathbf{n}};\phi) Q(\hat{\mathbf{n}}) + B^U_{ij}(\hat{\mathbf{n}};\phi) U(\hat{\mathbf{n}}) \Big] d^2\hat{\mathbf{n}} + n_{ij}(\phi)$$

- Transfer function:

$$B^X_{ij}(\hat{\mathbf{n}};\phi) = \frac{1}{\Omega_{ij}} A^a_i(\hat{\mathbf{n}};\phi) A^{b*}_j(\hat{\mathbf{n}};\phi) \mathcal{P}^X_{ab}(\hat{\mathbf{n}}) e^{2\pi i \hat{\mathbf{n}} \cdot \mathbf{u}_\alpha(\phi)}$$

- m-mode map:

$$V_{ij;m} = \sum_l \left[B^T_{ij;lm} a^T_{lm} + B^E_{ij;lm} a^E_{lm} + B^B_{ij;lm} a^B_{lm} \right] + n_{ij;m}$$

Interferometric Imaging

- Traditional imaging is based around the 2D Fourier Transform approximation to the measurement equation (only valid on small patches instantaneously)
- Use a series of steps to relax this approximation and increase field of view (w-projection, mosaicking, A-projection)
  - eg. w-term. From non coplanarity of array and sky. Solve by iteratively deconvolving the effects

$$V = \int dx dy A^2(x, y) e^{2\pi i (ux + vy + w\sqrt{1 - x^2 - y^2})} I(x, y)$$

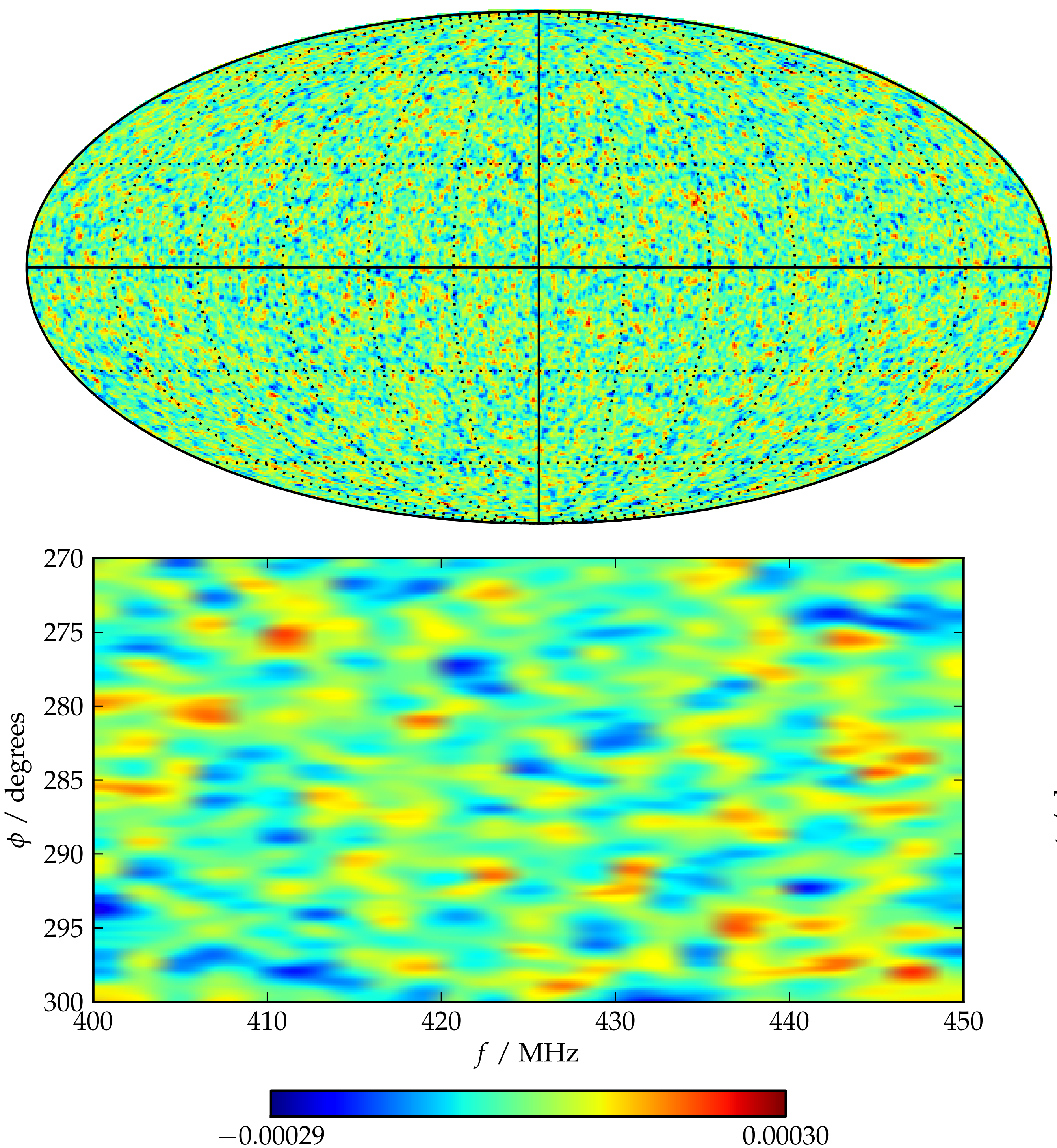
m-mode Imaging

- For our restricted domain (transit telescopes), we can solve the problem exactly.
- Measurement is linear mapping:

$$\mathbf{v} = \mathbf{B}\,\mathbf{a} + \mathbf{n}\ .$$

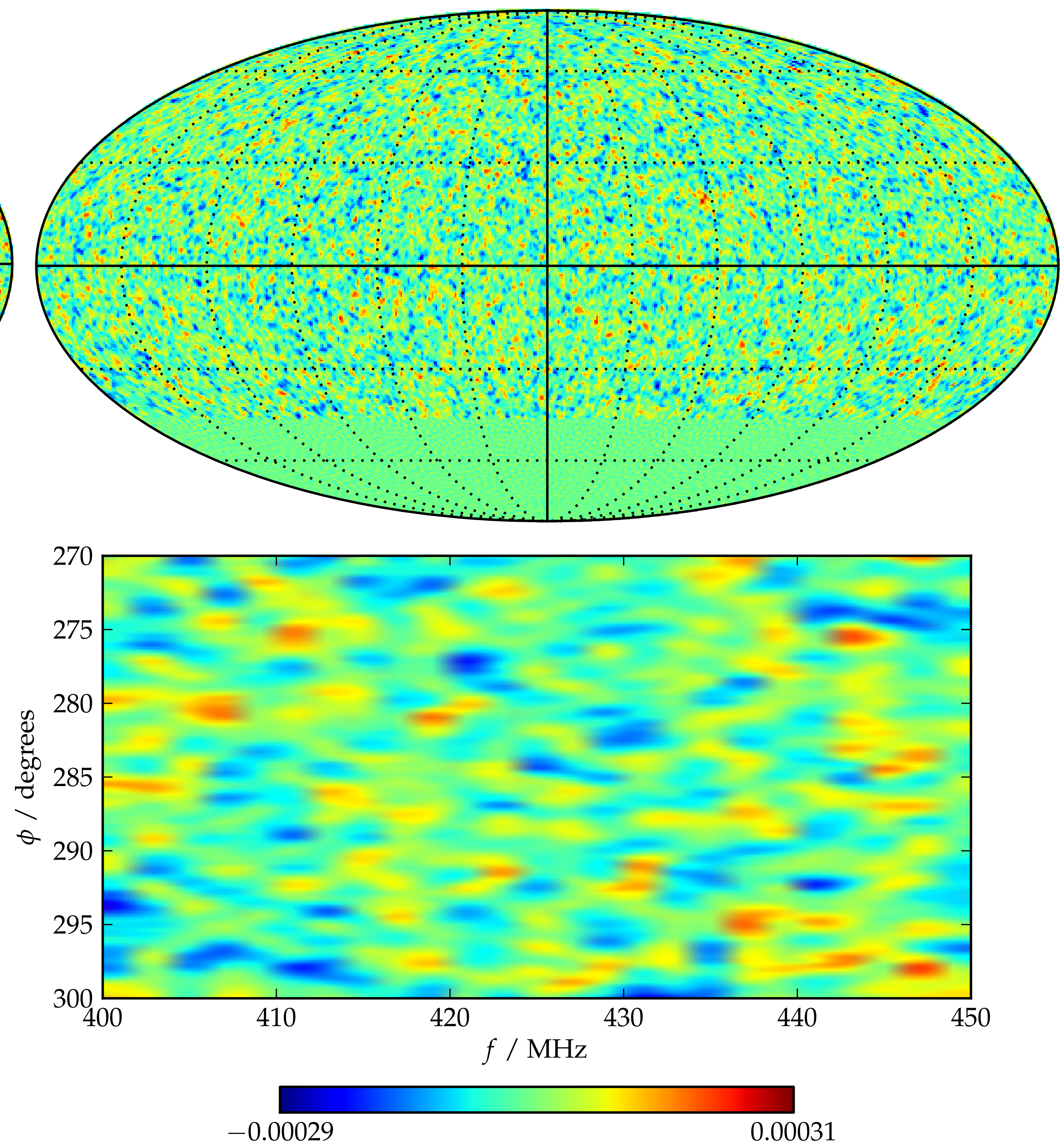
- How do we make an image of the sky? Use standard tools of signal processing:
  - Pseudo-inverse to solve and regularize *(maximum likelihood)*
  - Wiener Filter *(Bayesian expectation)*
- Conceptually straightforward. Deals naturally with all full sky effects, polarisation etc.

Simulated



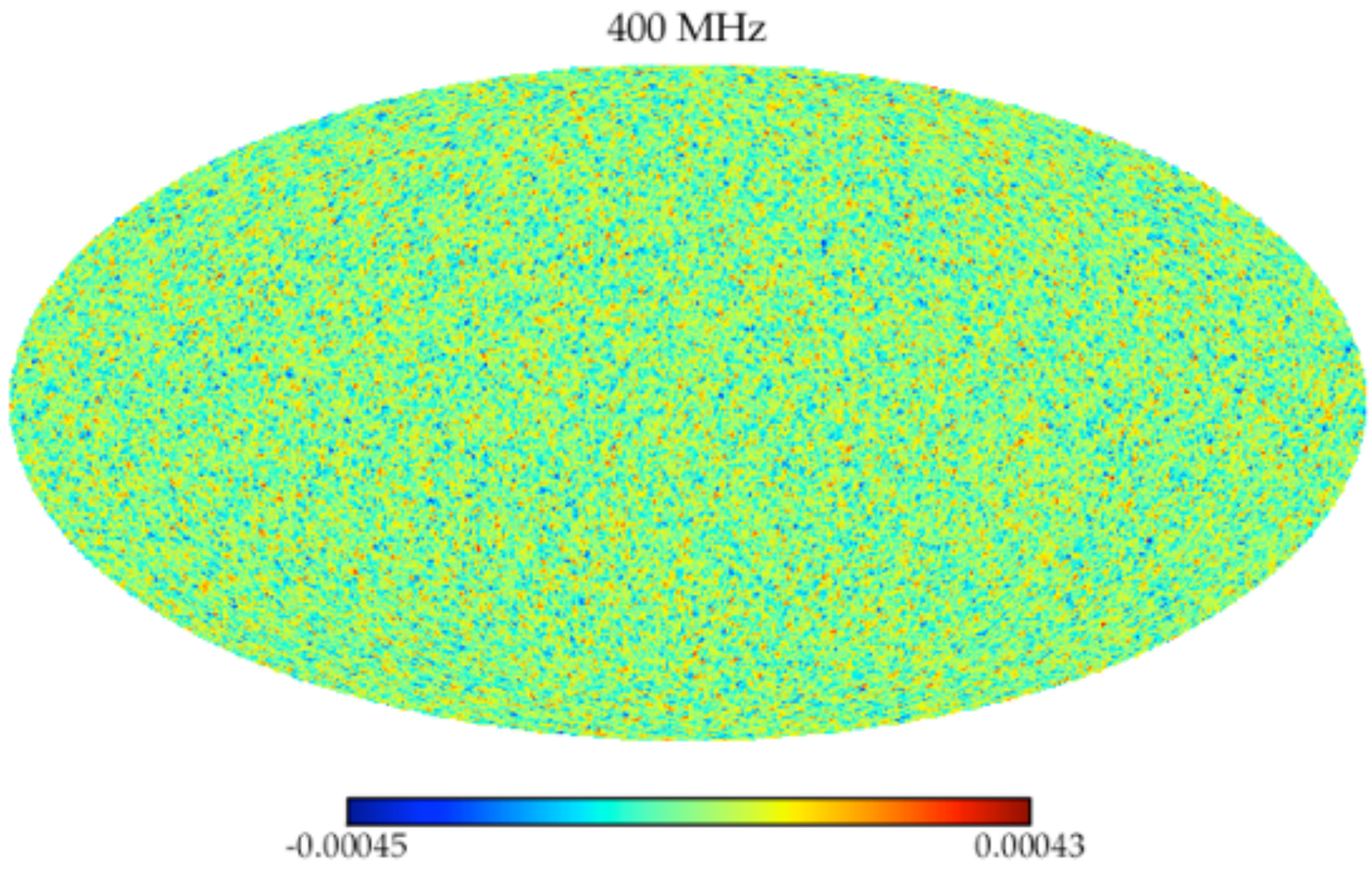
Observed map

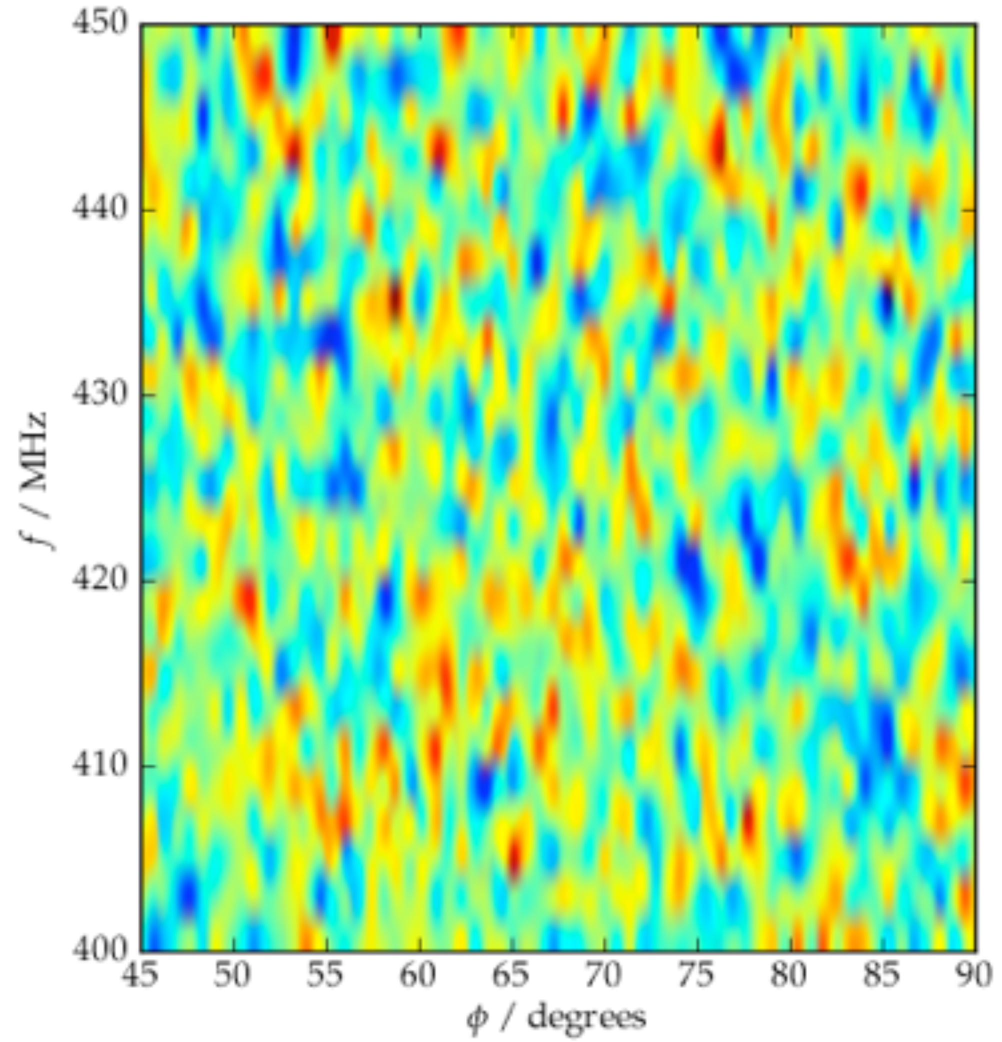
$$\hat{\mathbf{a}} = \left(\mathbf{N}^{-\frac{1}{2}}\mathbf{B}\right)^{+}\mathbf{N}^{-\frac{1}{2}}\mathbf{v}$$



2x15m wide cylinders, 60 feeds, 0.25m spacing 400-600 MHz

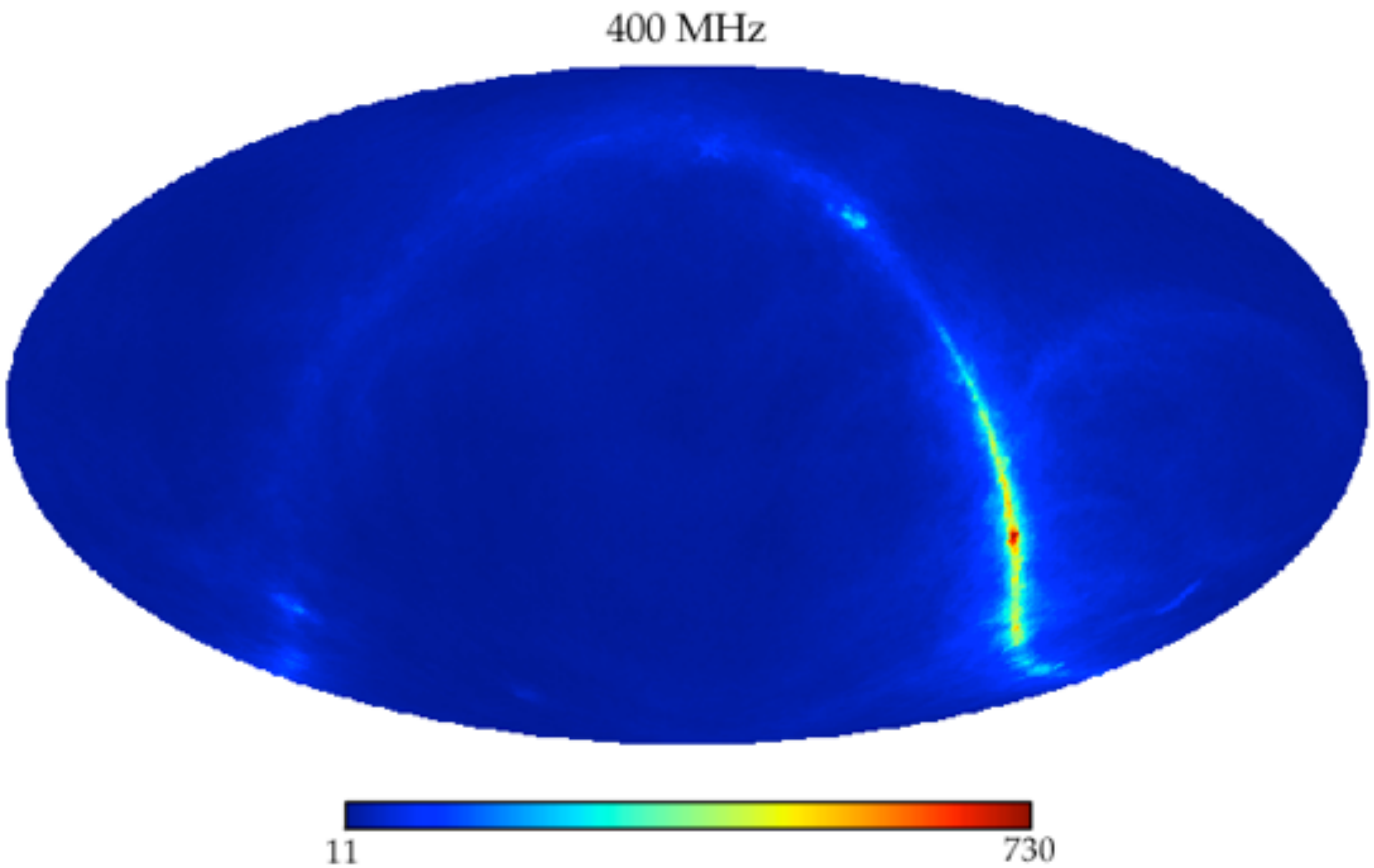
Foreground Challenges

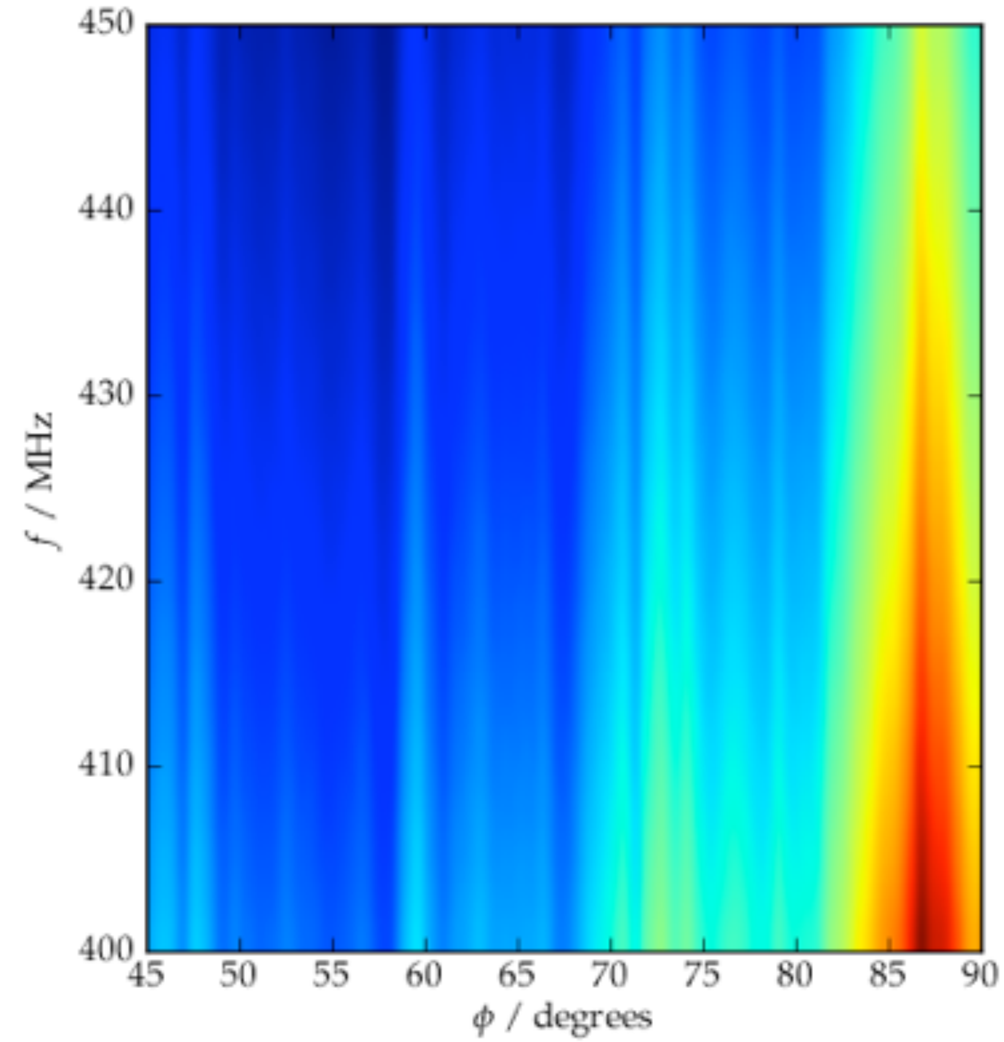




Cosmological 21cm Signal ~ 1mK

Foreground Challenges





Galaxy: up to 700K

Foreground Removal

- Spectral smoothness allows separation of 21cm
  - Measure components and model (Liu, Dillon etc.)
  - Power spectrum removal (Foreground wedge)
  - Delay-space filtering (Parsons et al. 2012)
- Most methods have difficulties:
  - *Mode mixing* of angular and frequency fluctuations by frequency-dependent beams (esp. interferometers)
  - *Robustness* Biasing introduced if foreground model poorly understood (esp. non-gaussianities)
  - *Statistical Optimality* Need to keep track of transformations on statistics, for optimal PS estimation
  - *Polarisation leakage* mixes fluctuations from polarised foreground

Signal to Noise Eigenmodes

- Old CMB idea - E/B mode separation (Bunn et al. 2003)
- An 'optimal' treatment - m-modes makes it feasible.
- Construct the covariances of the signal and foregrounds in the measured basis

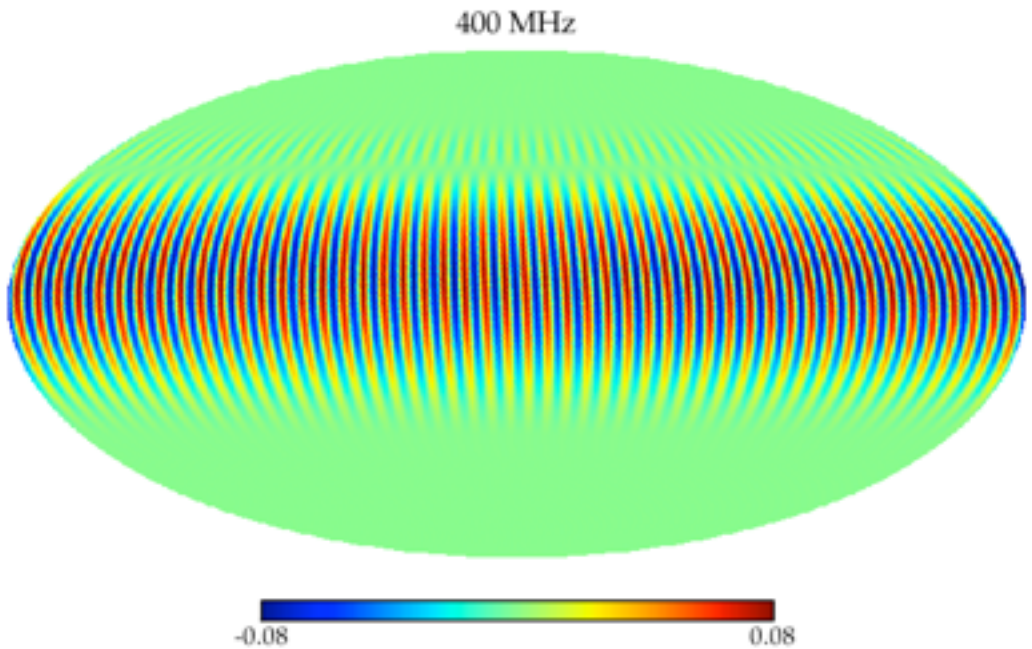
$$\mathbf{S} = \left\langle \mathbf{s}\mathbf{s}^\dagger \right\rangle = \mathbf{B} \left\langle \mathbf{a}_s^* \mathbf{a}_s^T \right\rangle \mathbf{B}^\dagger \qquad \mathbf{F} = \mathbf{B} \left\langle \mathbf{a}_f \mathbf{a}_f^\dagger \right\rangle \mathbf{B}^\dagger$$

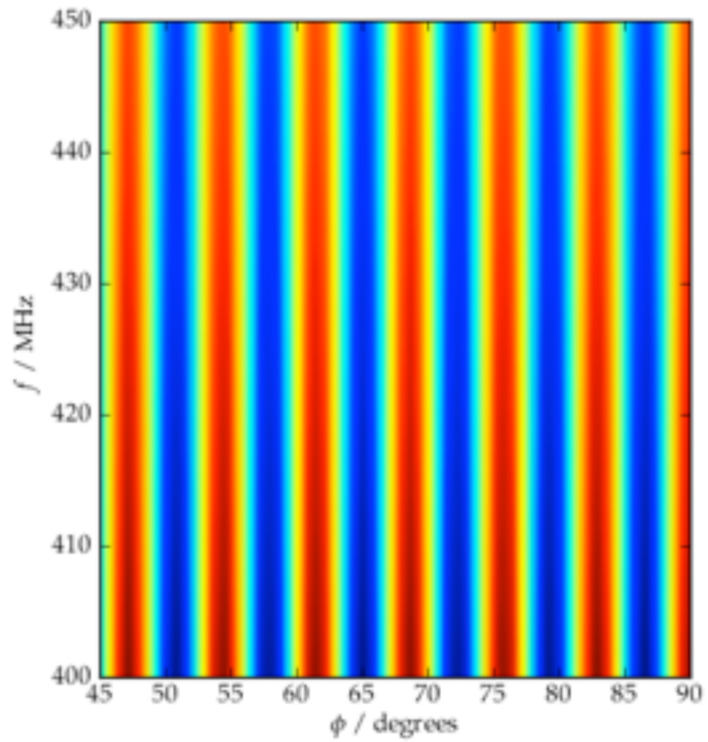
- Jointly diagonalise both (eigenvalue problem)

$$\mathbf{S}\mathbf{x} = \lambda \mathbf{F}\mathbf{x}$$

- Gives a new, uncorrelated basis. Corresponding eigenvalue gives the expected signal to foreground power ratio.

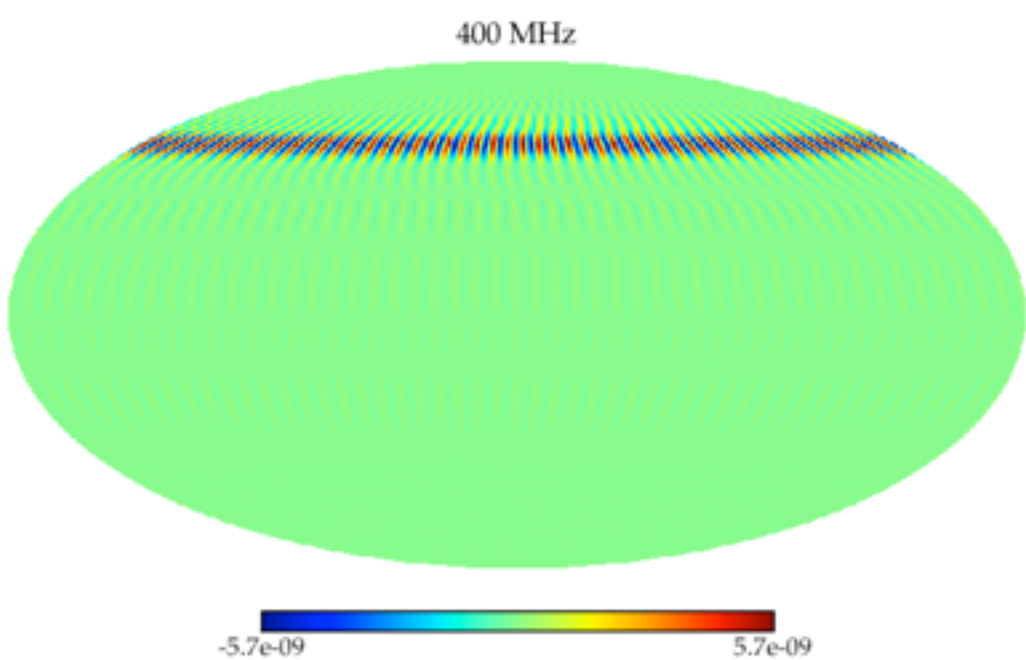
Most foreground

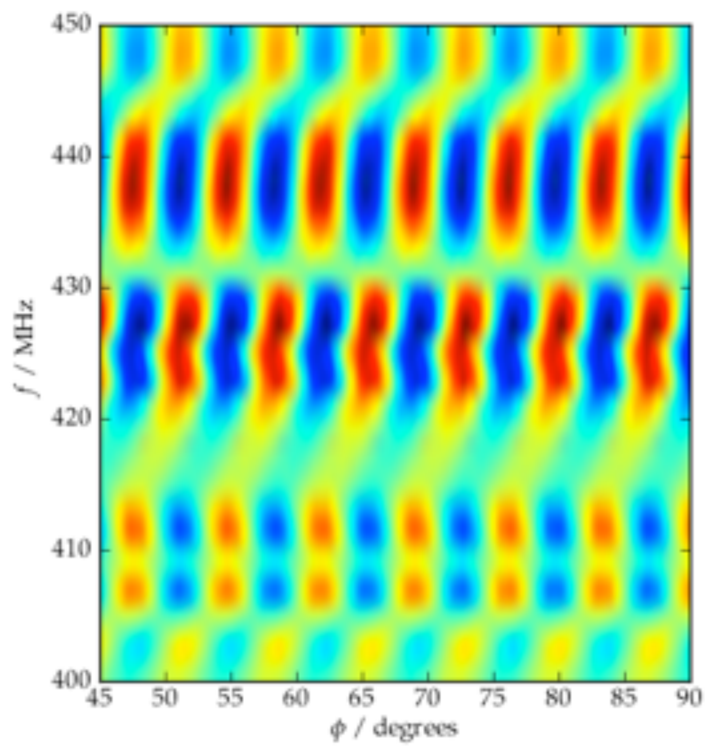




smooth in freq

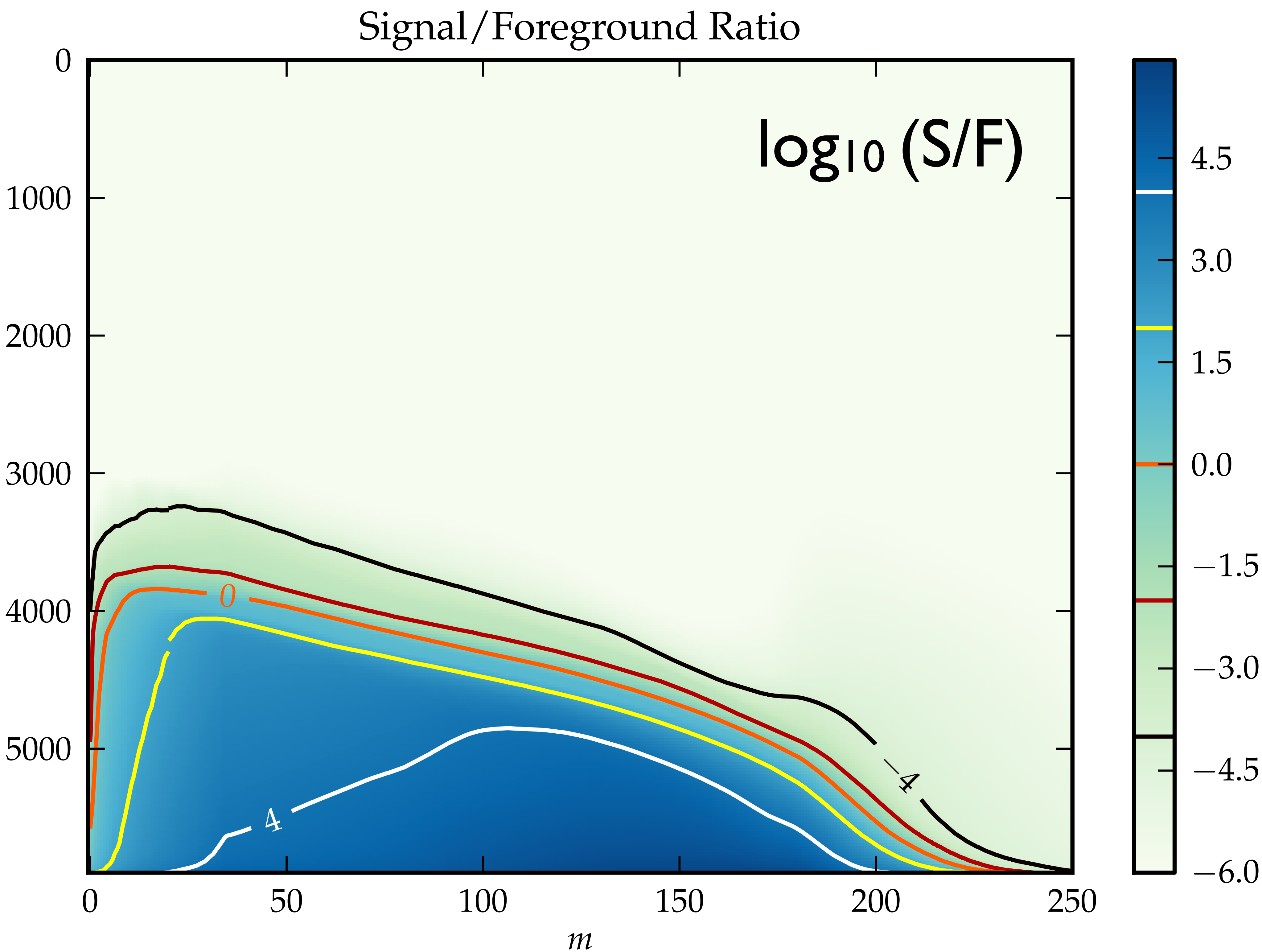
Most signal





oscillates in freq

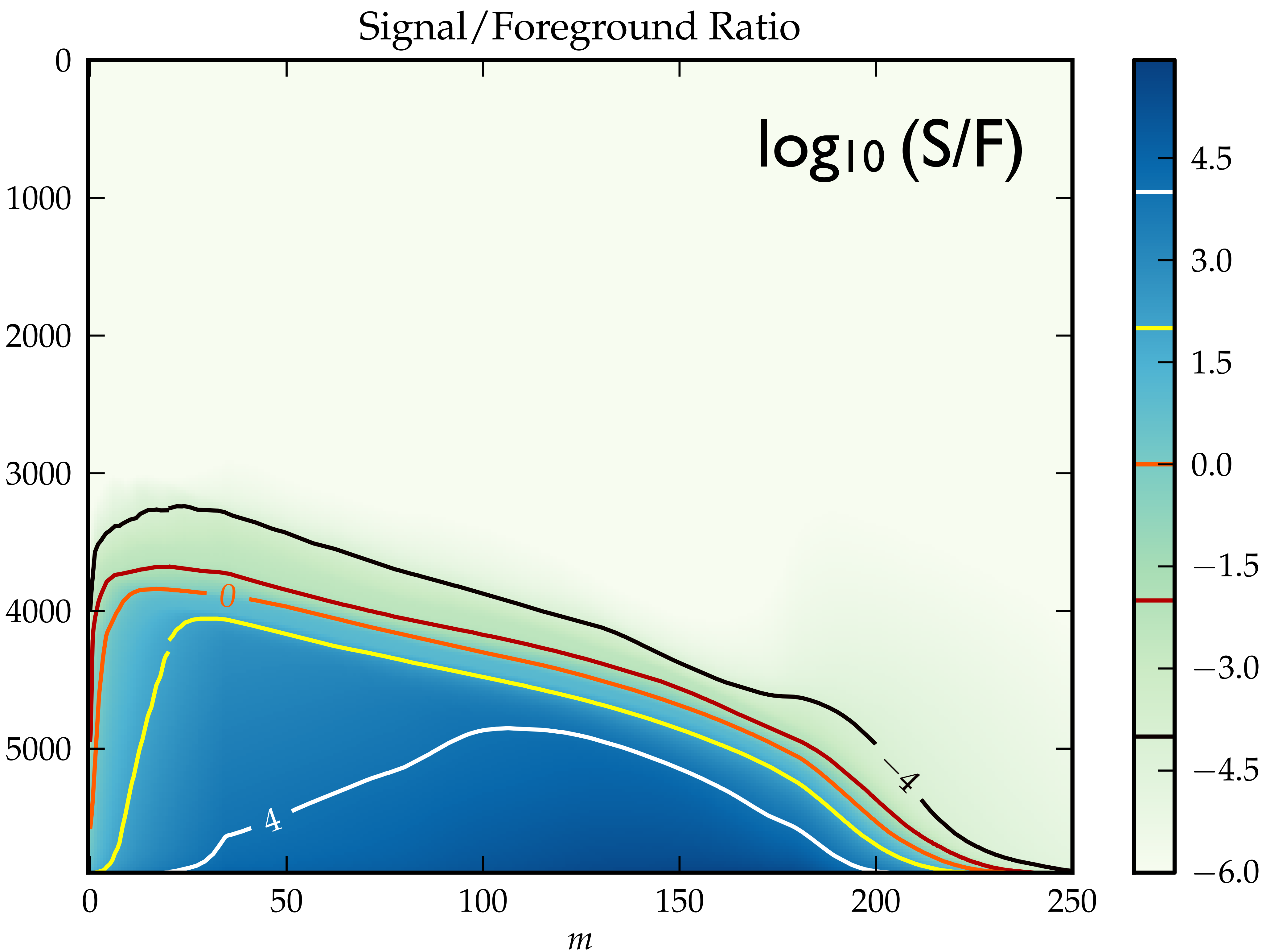
Signal/Foreground Ratio



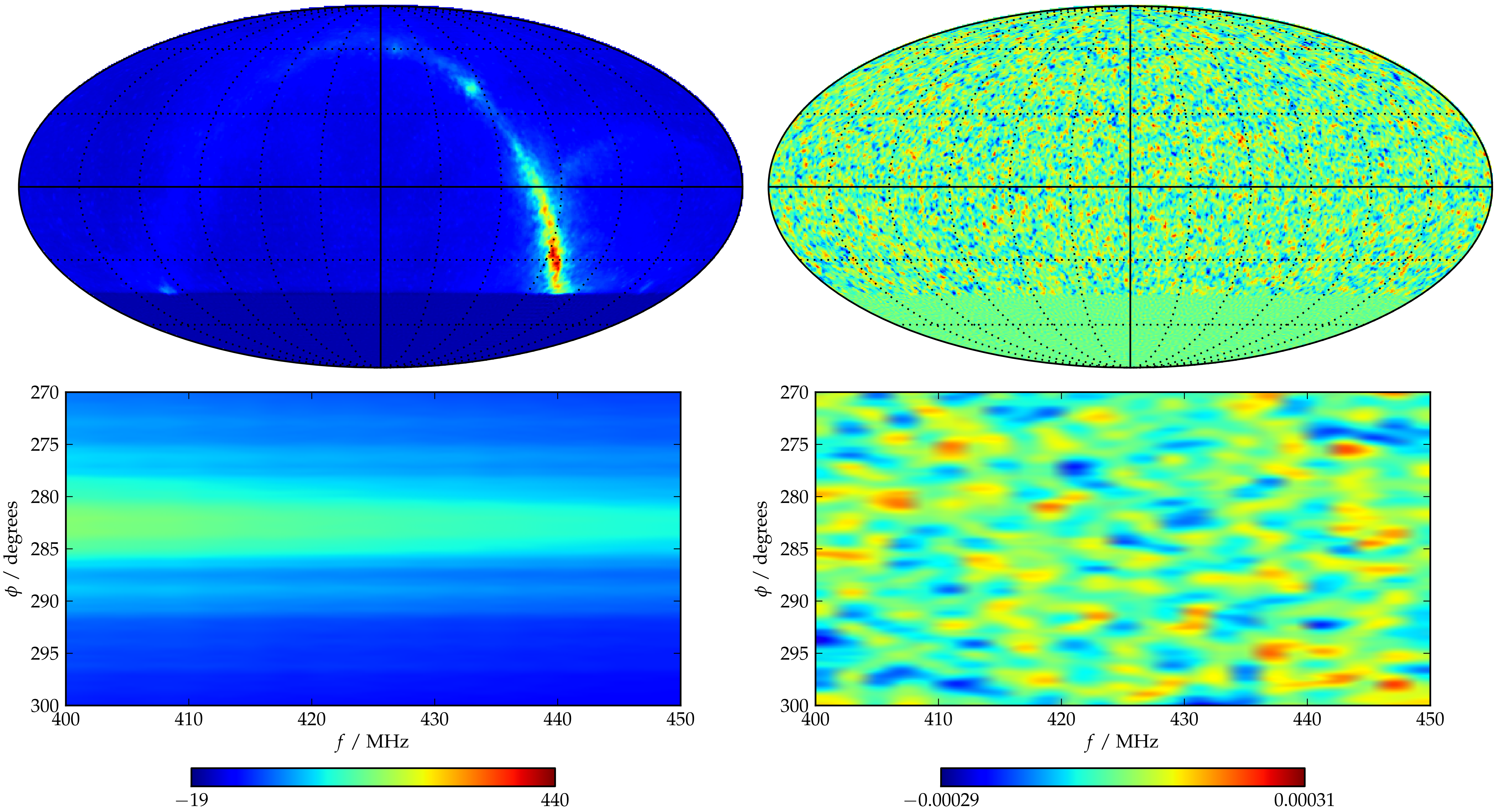
Foreground Removal with S/N modes

- Foreground removal is performed by projecting out modes with low signal-to-foreground ratio.
- Robustness to model uncertainties by choosing a conservatively large threshold; we would prefer to increase our errors bars in order to remove bias.
- Addresses the previous problems
  - Analysis uses all measured data to avoid mode mixing.
  - Can be made arbitrarily robust - increase threshold for removal
  - Linear transformation in the data space, keeps track of statistics

Signal/Foreground Ratio

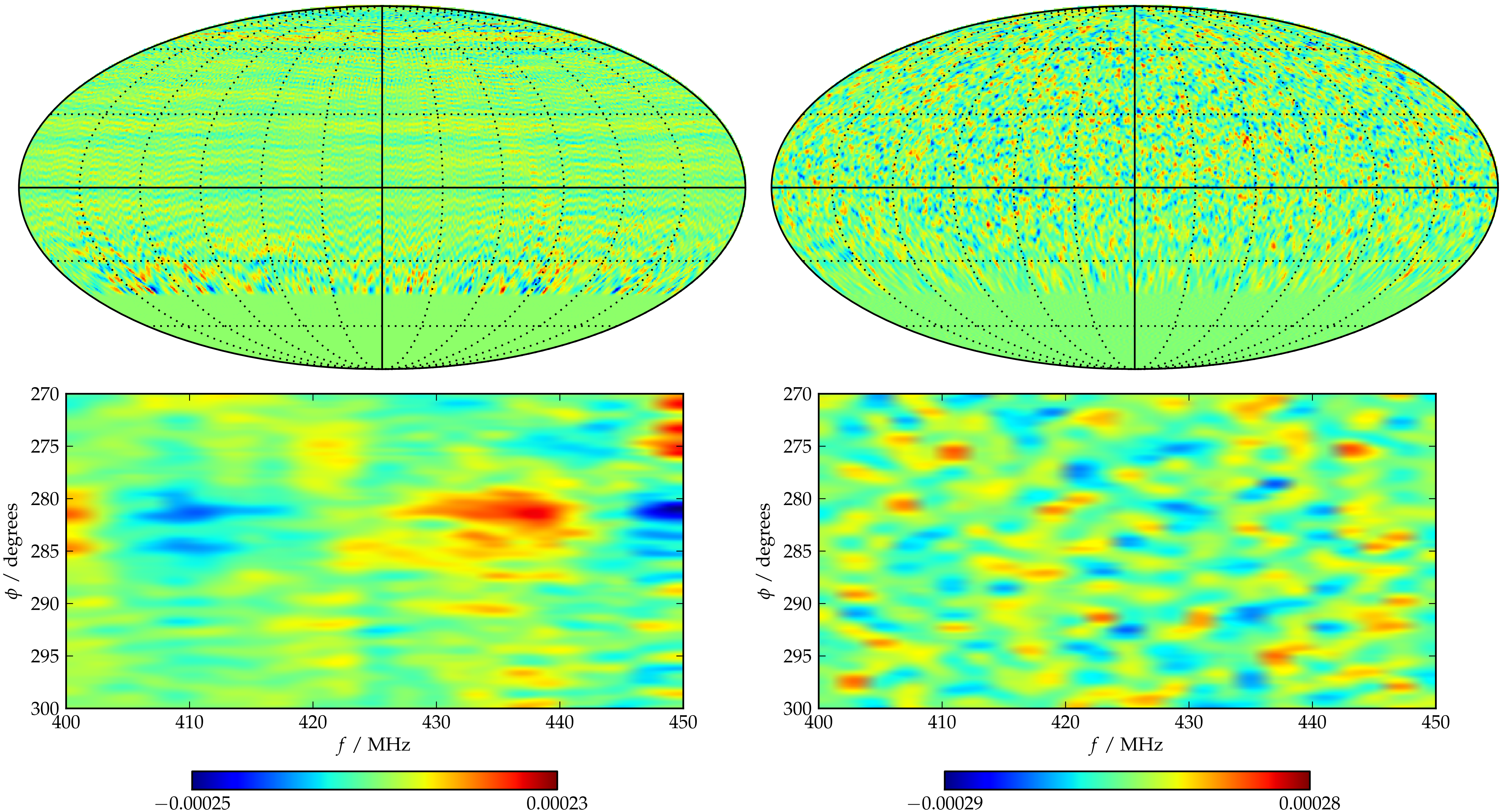


Observed sky *(from simulated time stream)*



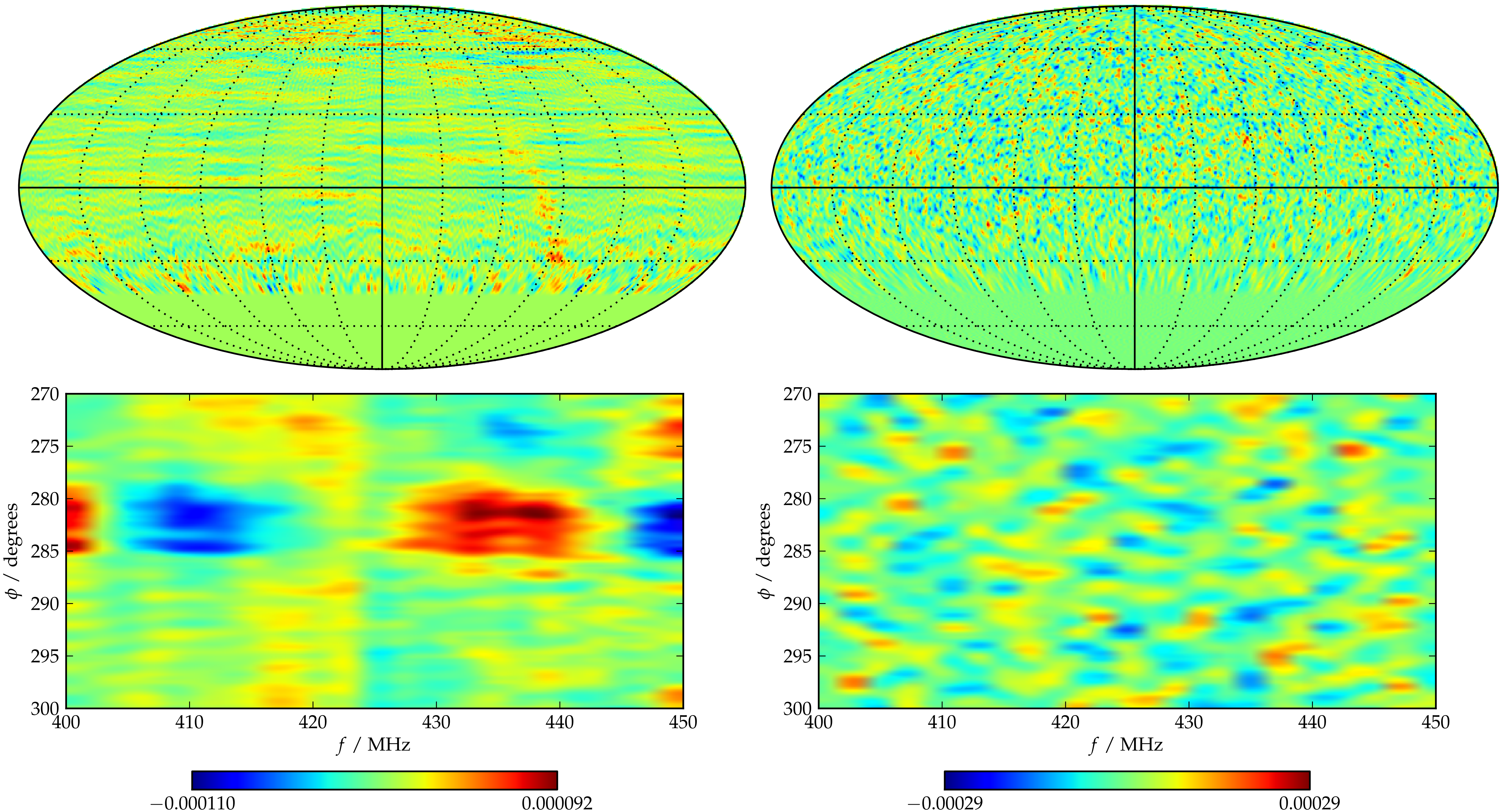
~10^6 x brighter

S/F > 0.01



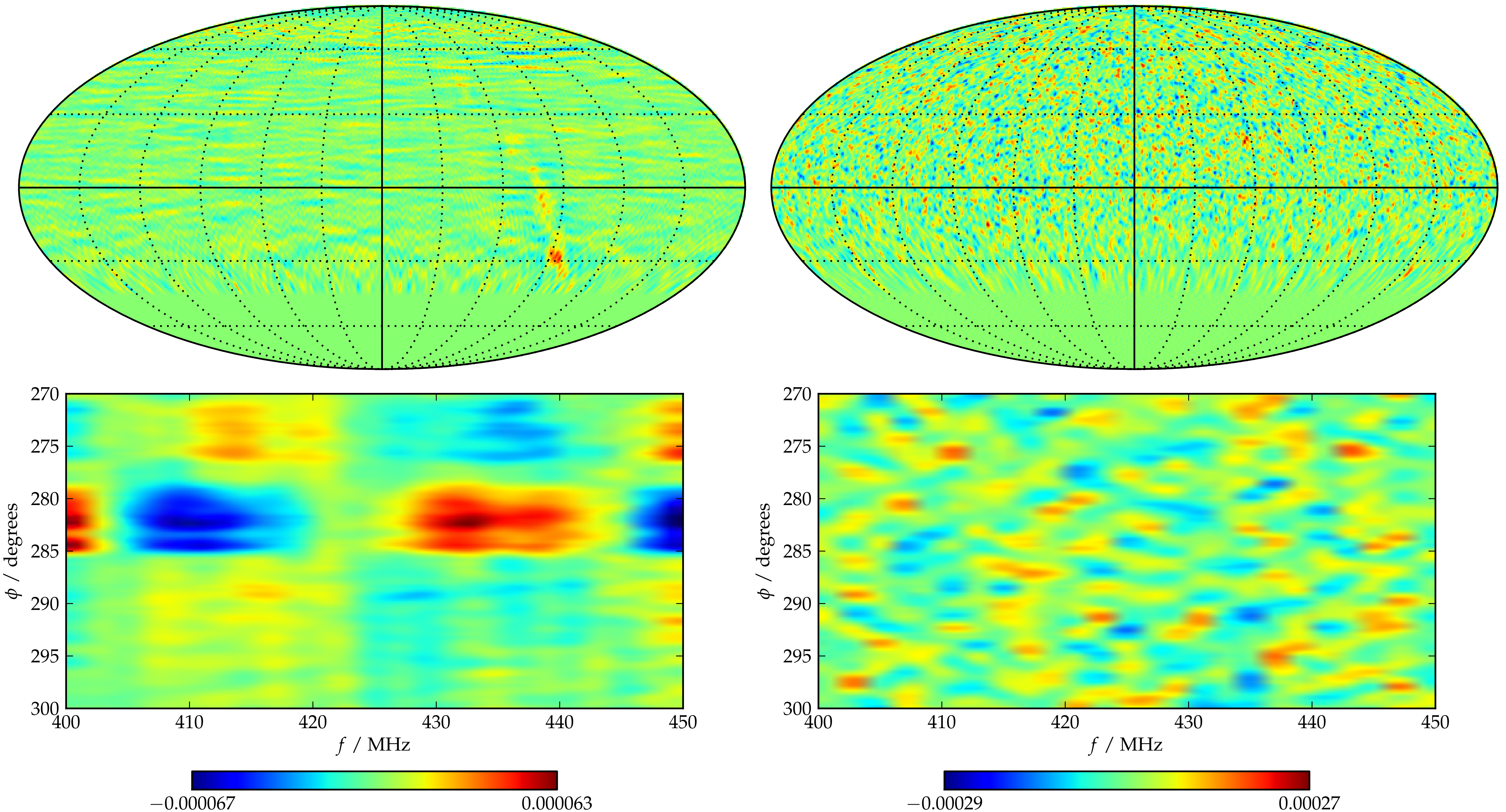
~ same brightness

S/F > 0.1



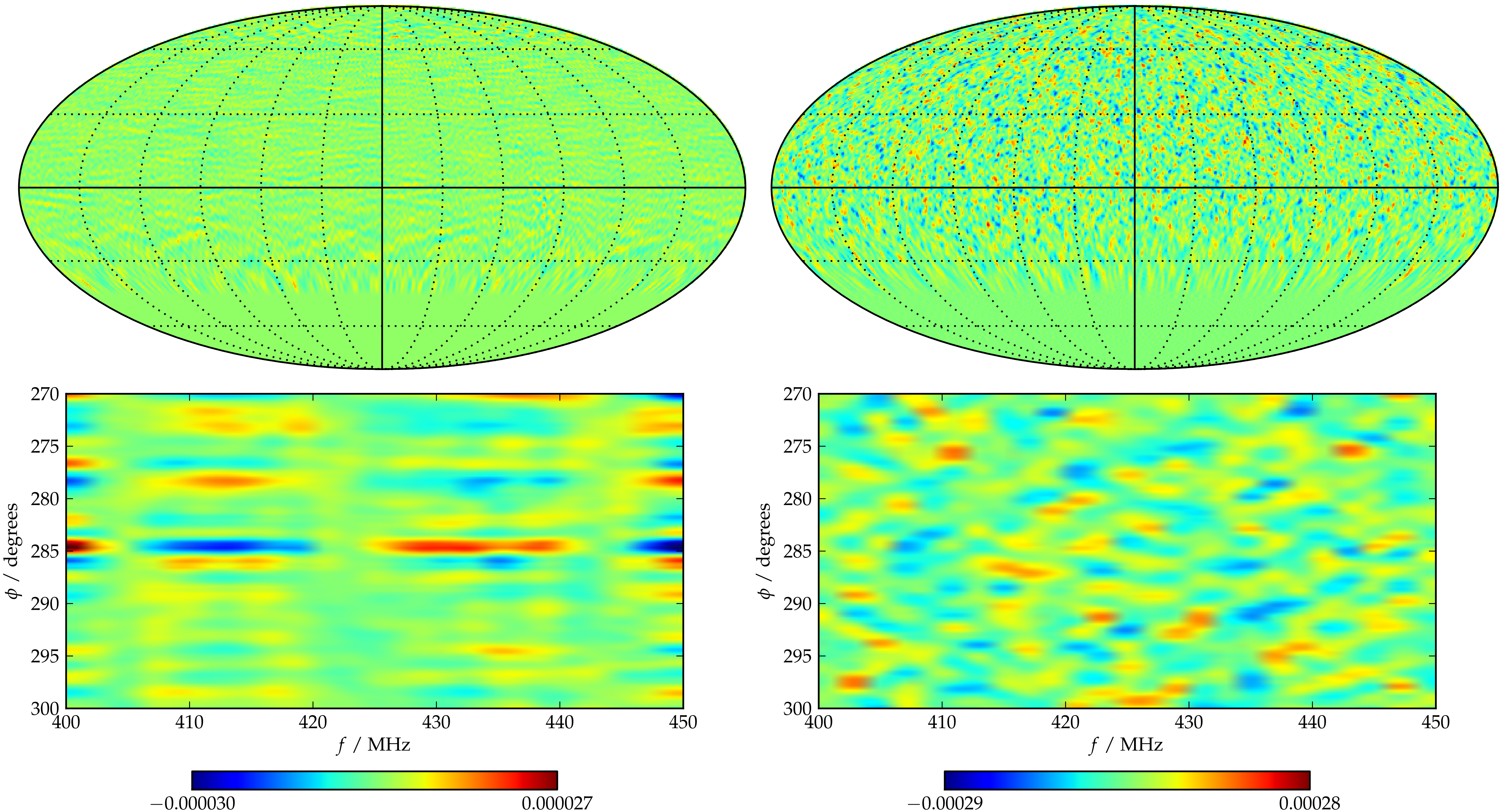
~ 3x dimmer

S/F > 1



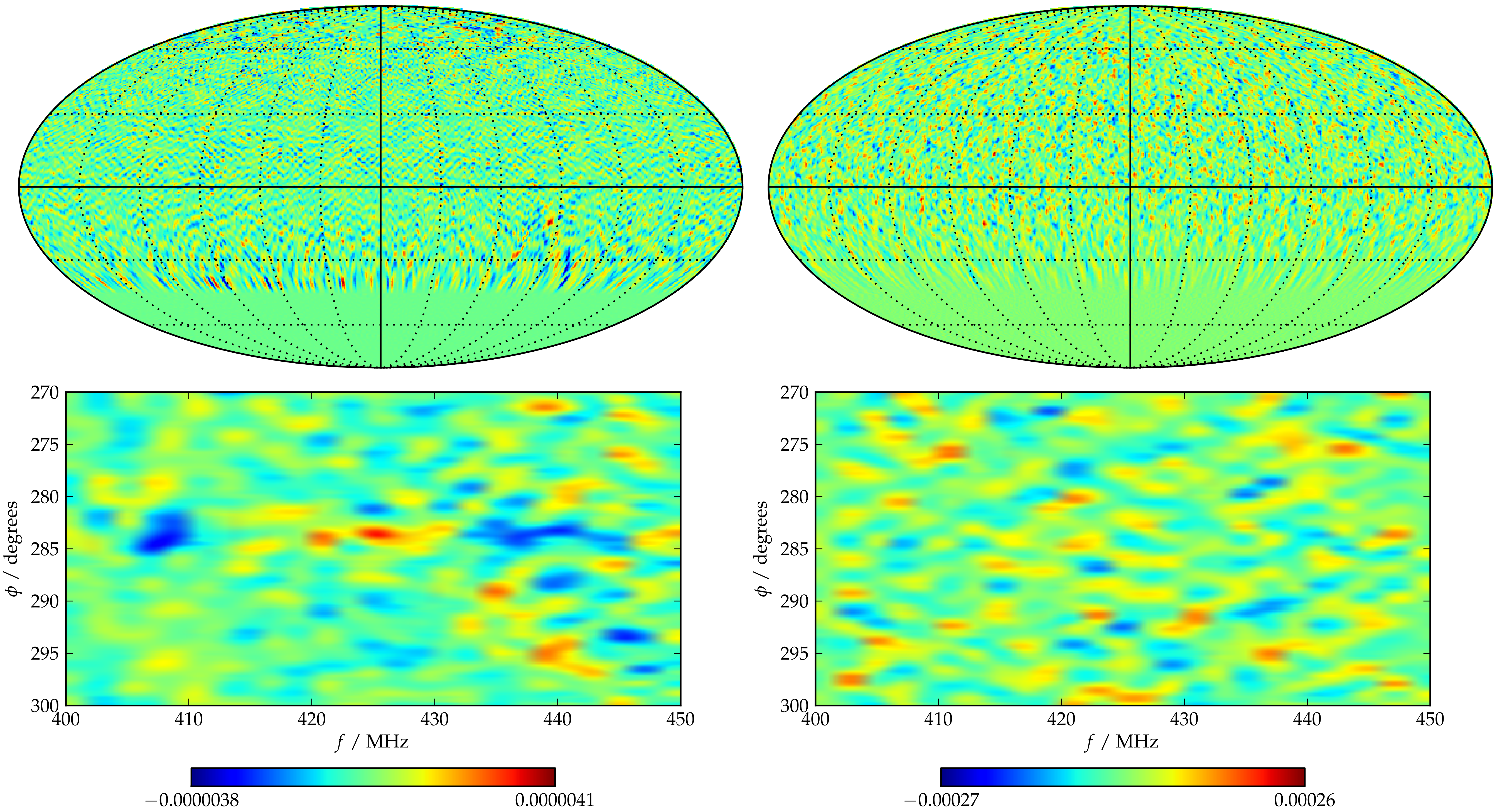
~ 5x dimmer

S/F > 10



~ 10x dimmer

S/F > 100



~ 70x dimmer

Summary

- Unconventional interferometers need unconventional data analysis:
  - m-mode transform is a promising method for analysing wide field interferometric data
  - Enable the use of Signal-to-Noise eigenmodes for foreground removal
  - Similar advantages for power spectrum estimation