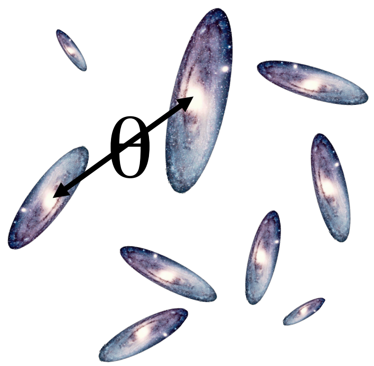
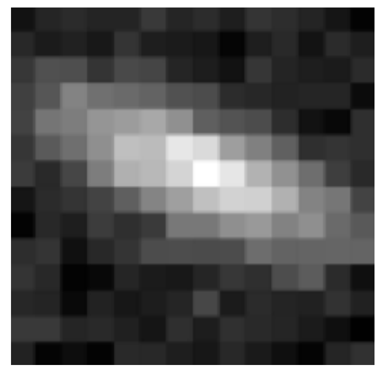
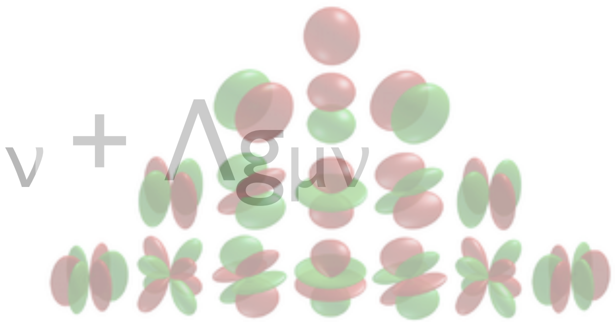




$$G_{\mu\nu}=8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$



Weak Gravitational Lensing

Tom Kitching
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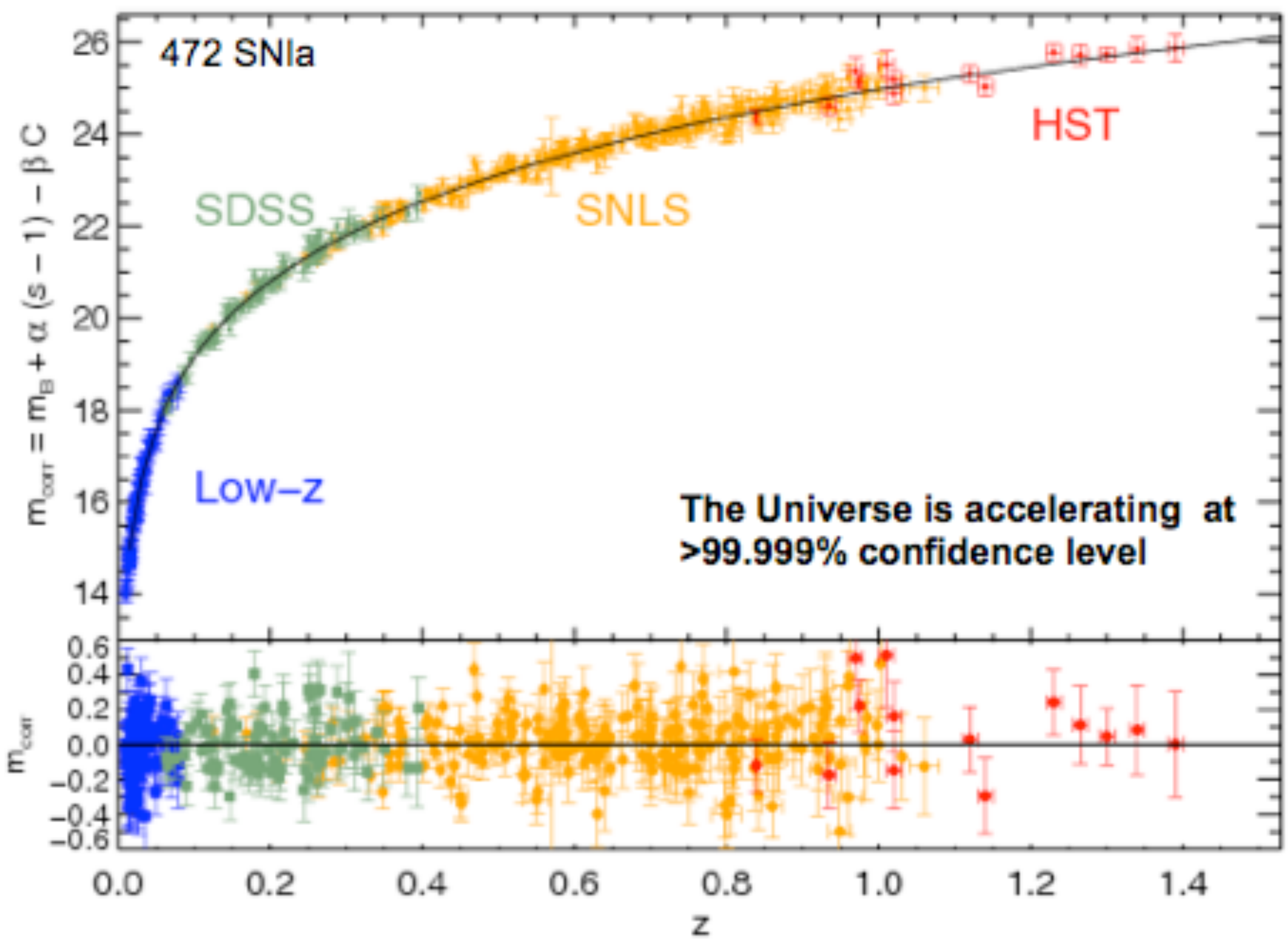
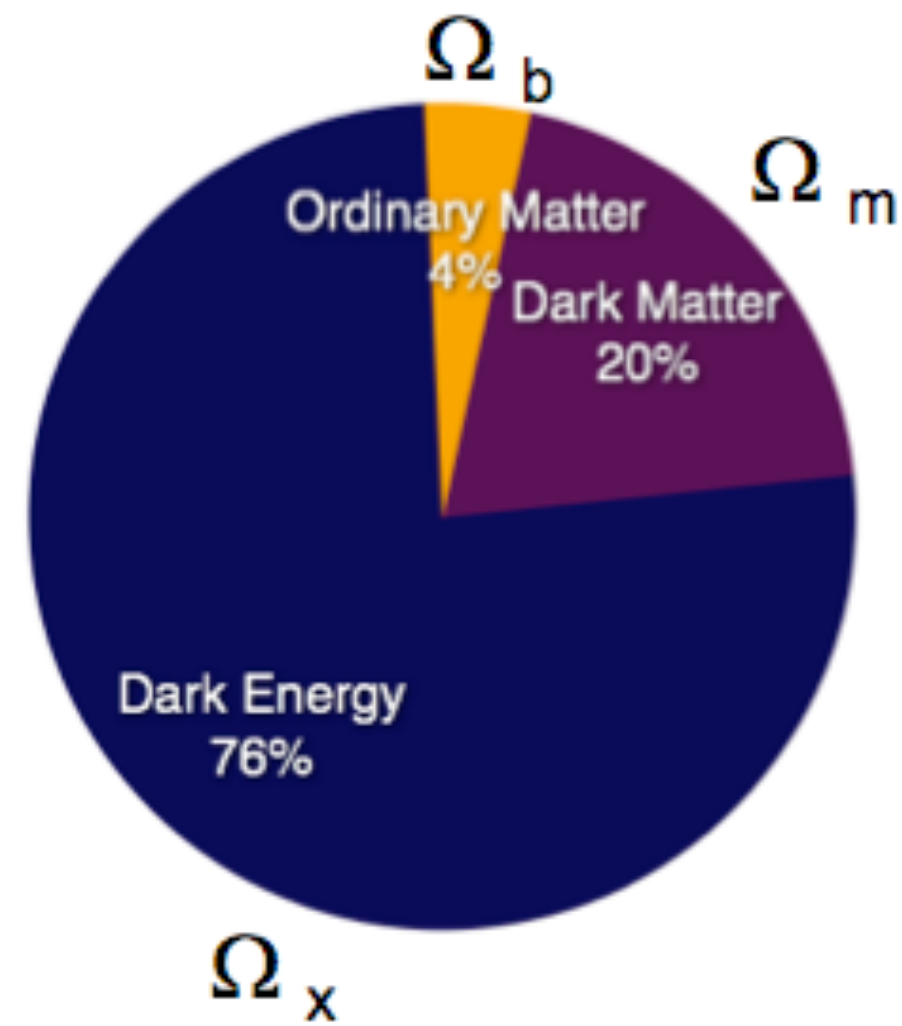
- Motivation
  - Cosmological Perspective and Dark Energy
  - Current and Future data
- My specific domain of application
  - Weak Gravitational Lensing
  - State-of-art
- Application to current data



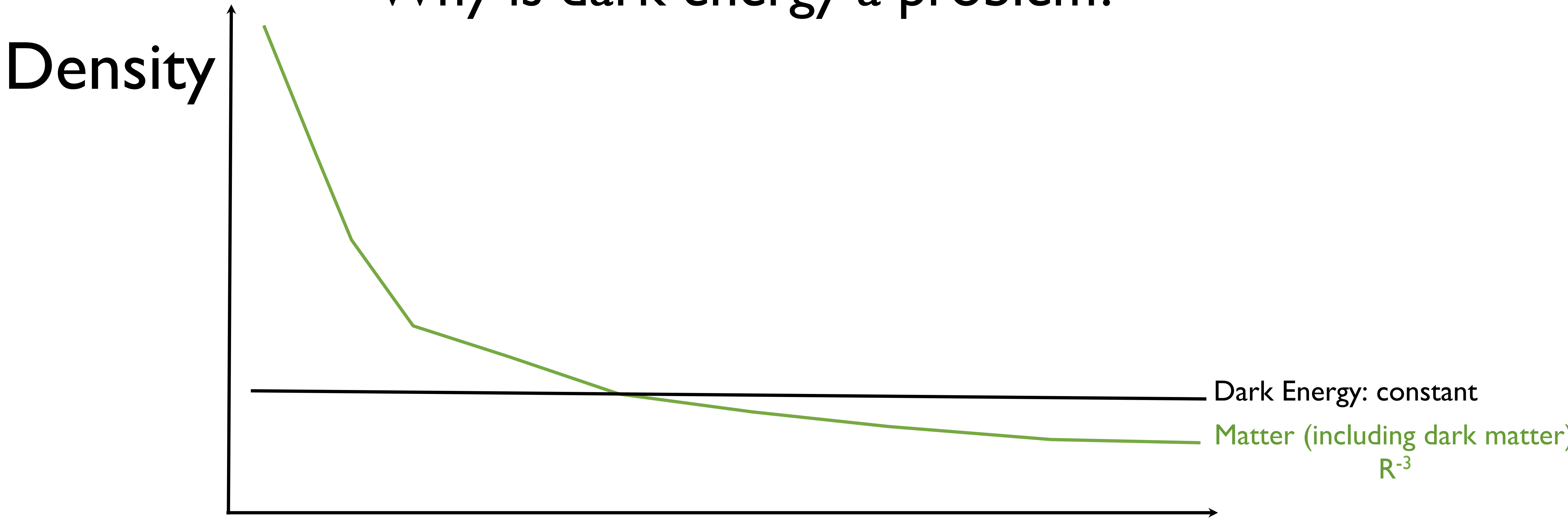
Conley et al 2011



An accelerated expansion

What is the Universe made of?

Why is dark energy a problem?



- Dark Energy is not matter or radiation
  - *Like* energy of the vacuum. Vacuum is sea of virtual particles but...
  - Prediction of energy density is 10^{120} orders of magnitude larger than what is observed
  - Is dark energy a symptom of new field or QFT being incorrect?
- Dark Energy is causing an accelerated expansion
  - *Like* anti-gravity - pushing the Universe apart
  - Is dark energy a symptom of general relativity being incorrect?

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- “Simplist” explantion is “cosmological constant”
- Like Newtons constant, G, simply a constant of nature

- But.... even in this case still have two problems
  - 1) Implies that somehow the vacuum energy is cancelled out exactly
    - $E_{vacuum} = 10^{120}$ (QFT) - 10^{120} (new effect) = 0
  - 2) Why does it have the value it does...?
    - Fine Tuning Problem

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$

- "Vacuum energy"

- But.... even in this case still have two problems
  - 1) Implies that somehow the vacuum energy is almost but not exactly cancelled out
    - $E_{vacuum} = 10^{120}$ (QFT) - 10^{119} (new effect) ~ 1
  - 2) Why?
    - Fine Tuning Problem
    - Selection from landscape of vacua…

$$\frac{\ddot{a}}{a} = -4\pi G\left(\frac{\rho}{3} + \frac{p}{c^2}\right)$$

Acceleration equation

Can define an equation of state $wc^2 = p/\rho$

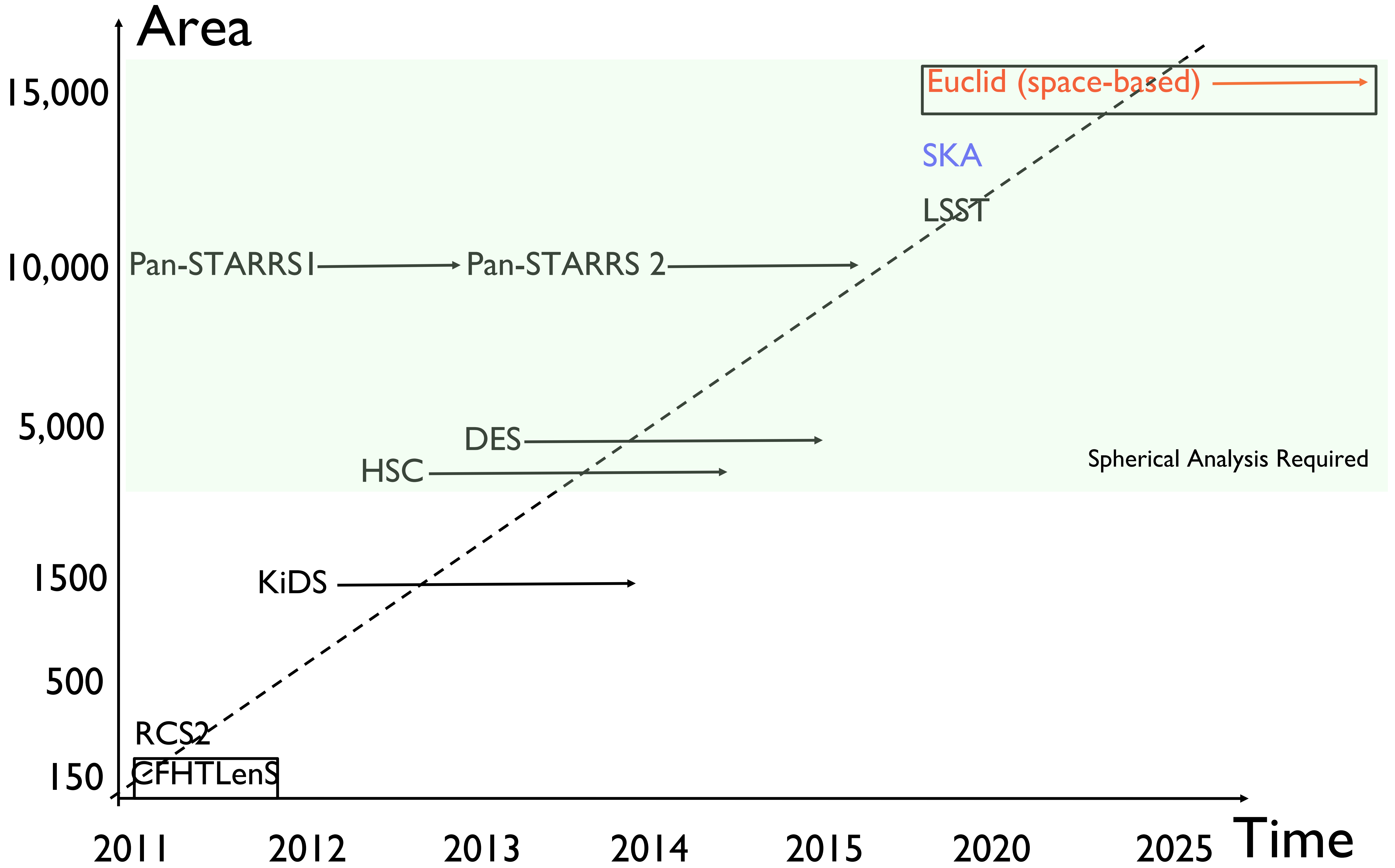
$$\frac{\ddot{a}}{a} = -4\pi G\, \rho(1/3+w)$$

So.. if $w<-1/3$ then $\ddot{a}>0$ and we have acceleration

Cosmological constant/vacuum energy would give $w \equiv -1$
Change gravity can (not always) generically give $w != -1$
Scalar (“higgs-like”) field can (not always) give time-varying $w(t\ ;\ z)$

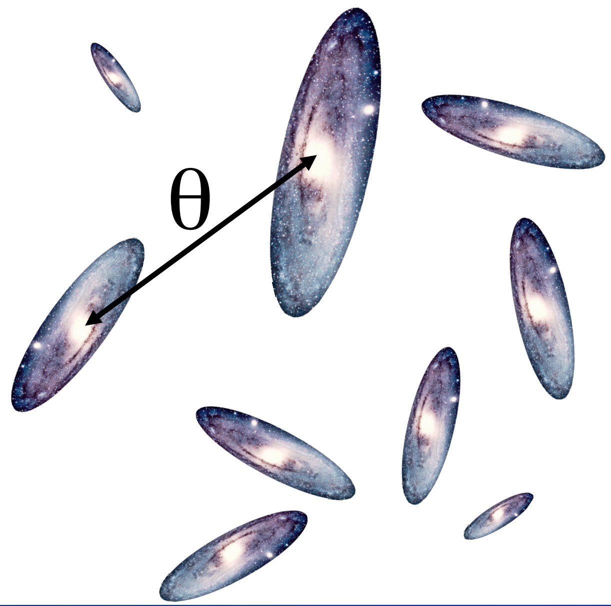
- There is a massive amount of investment in experiments to determine Dark Energy properties

- All of these use (will use) a technique called weak gravitational lensing

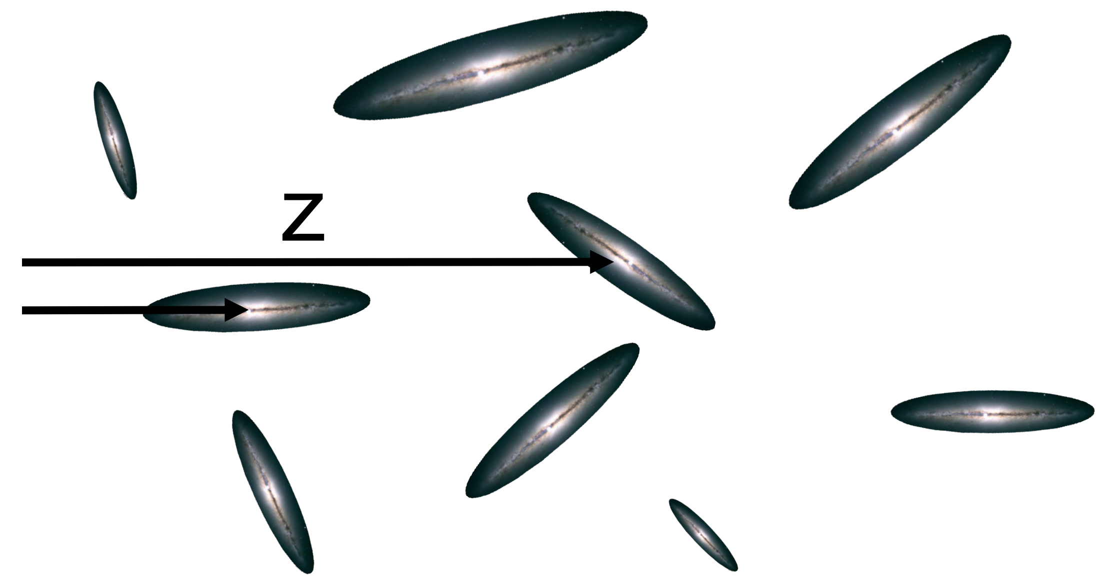


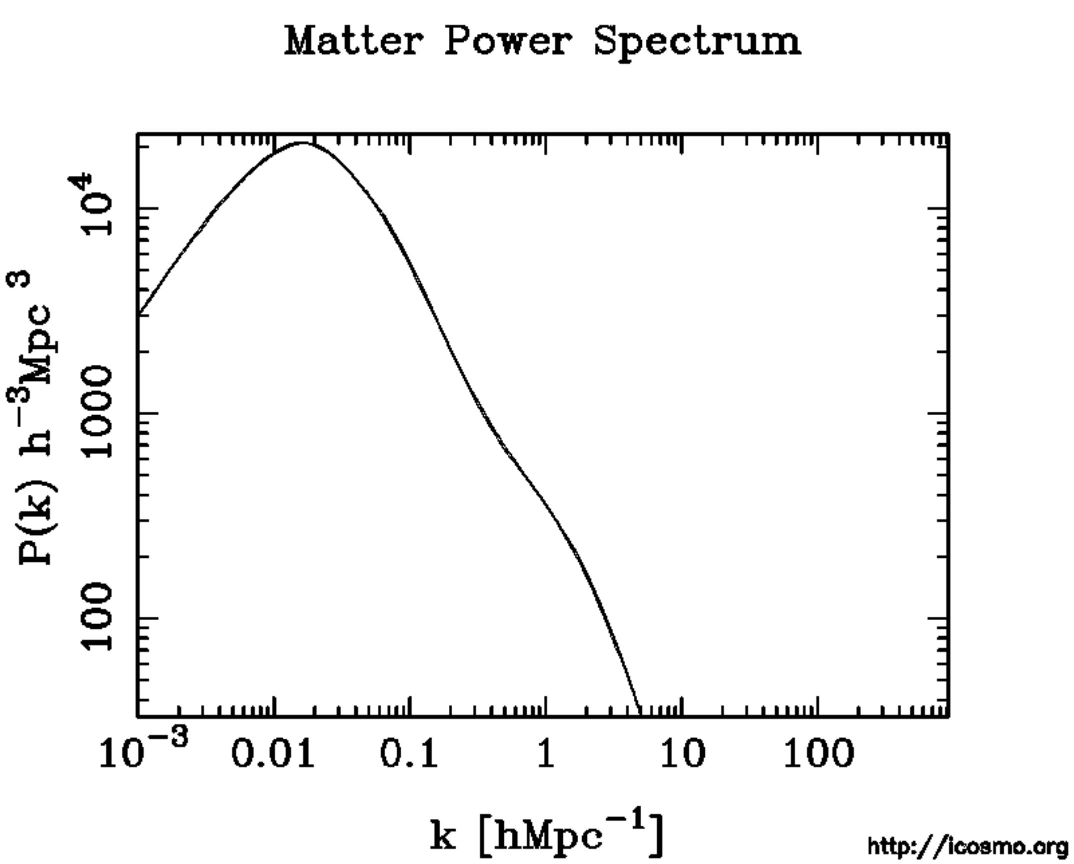
What can we observe?

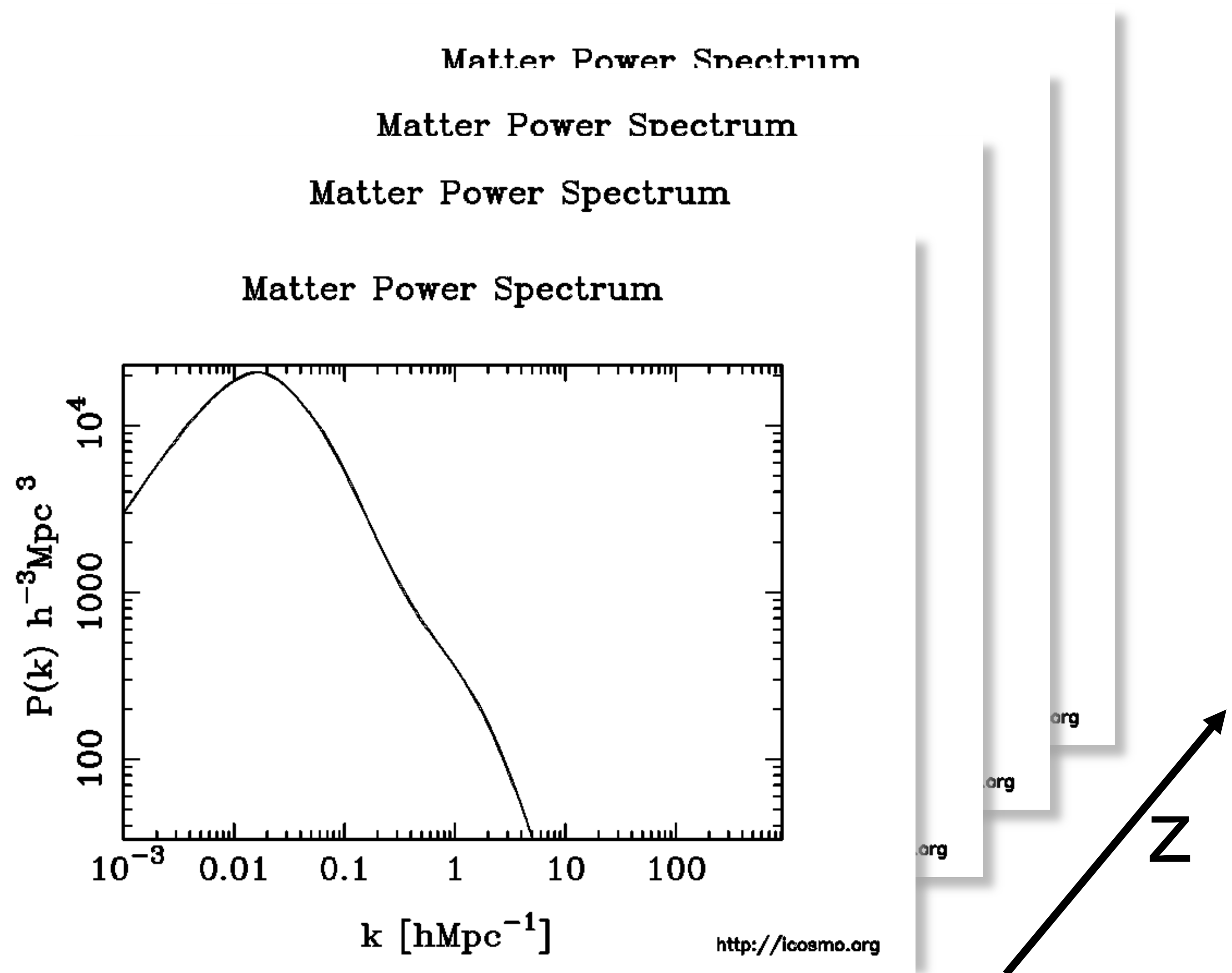
Angles



Wavelengths

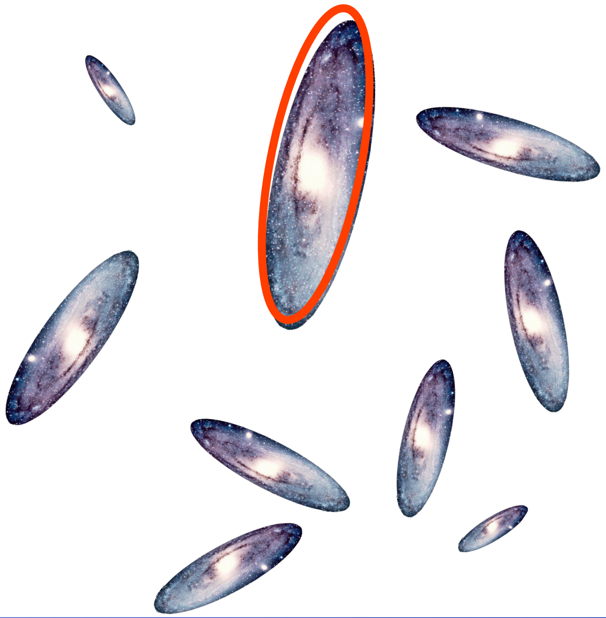


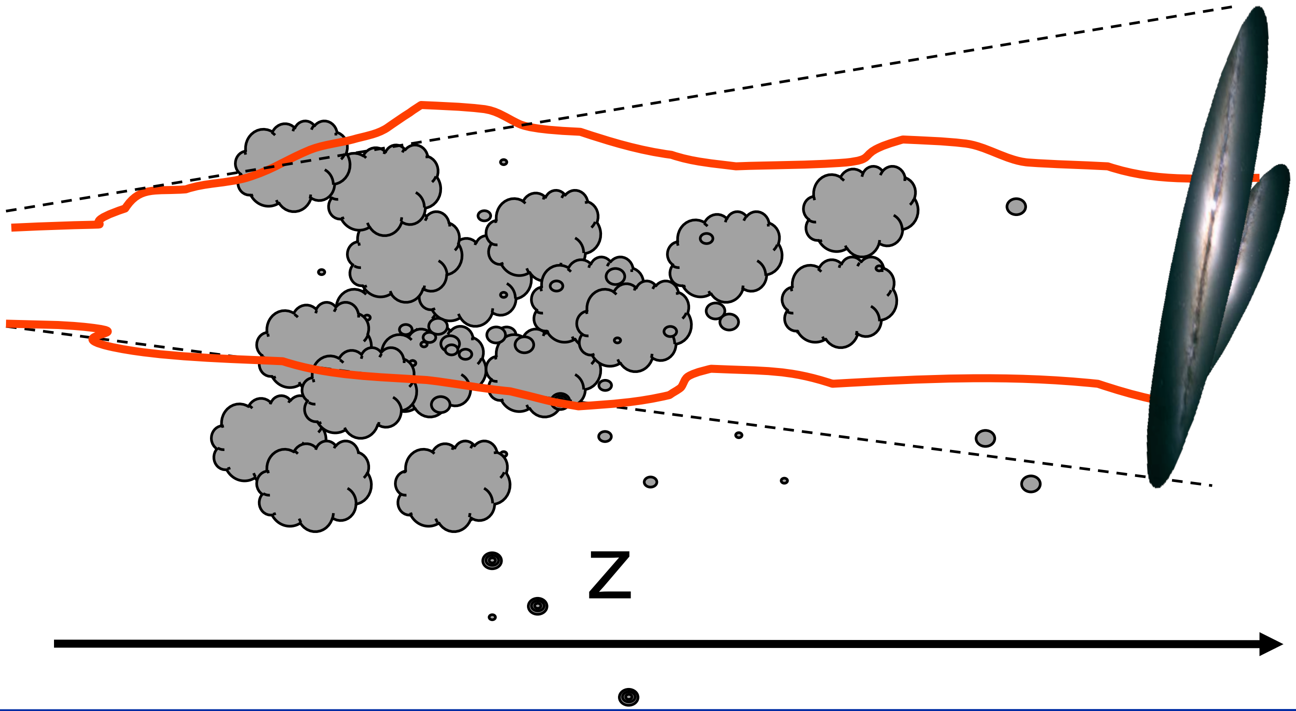


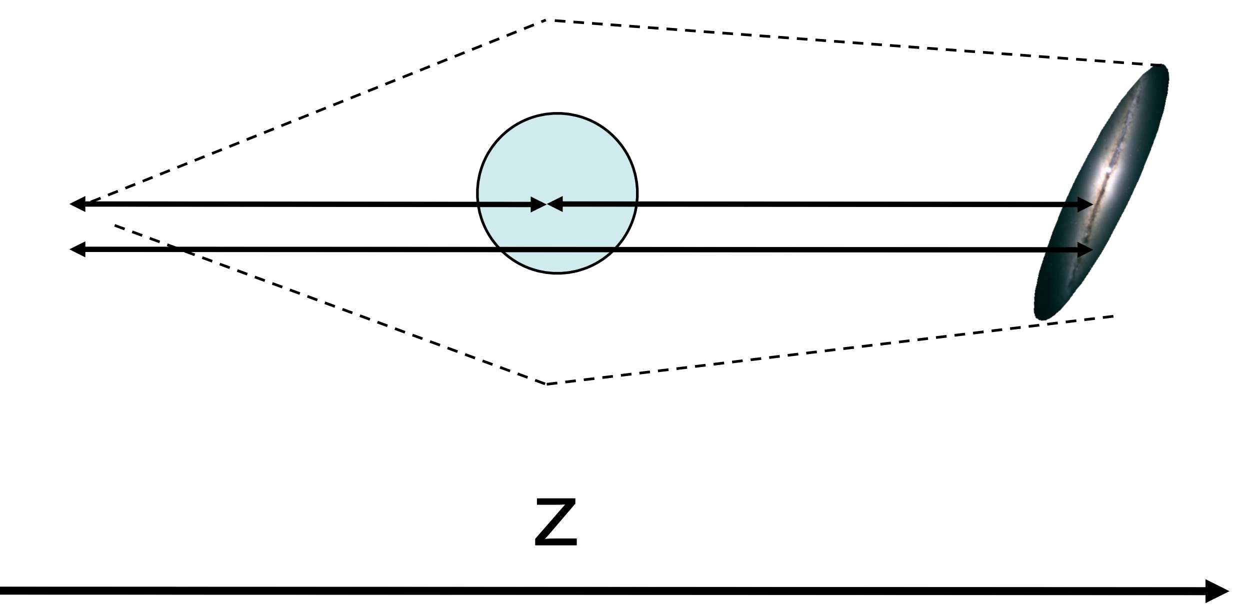


What can we observe?

Shapes!

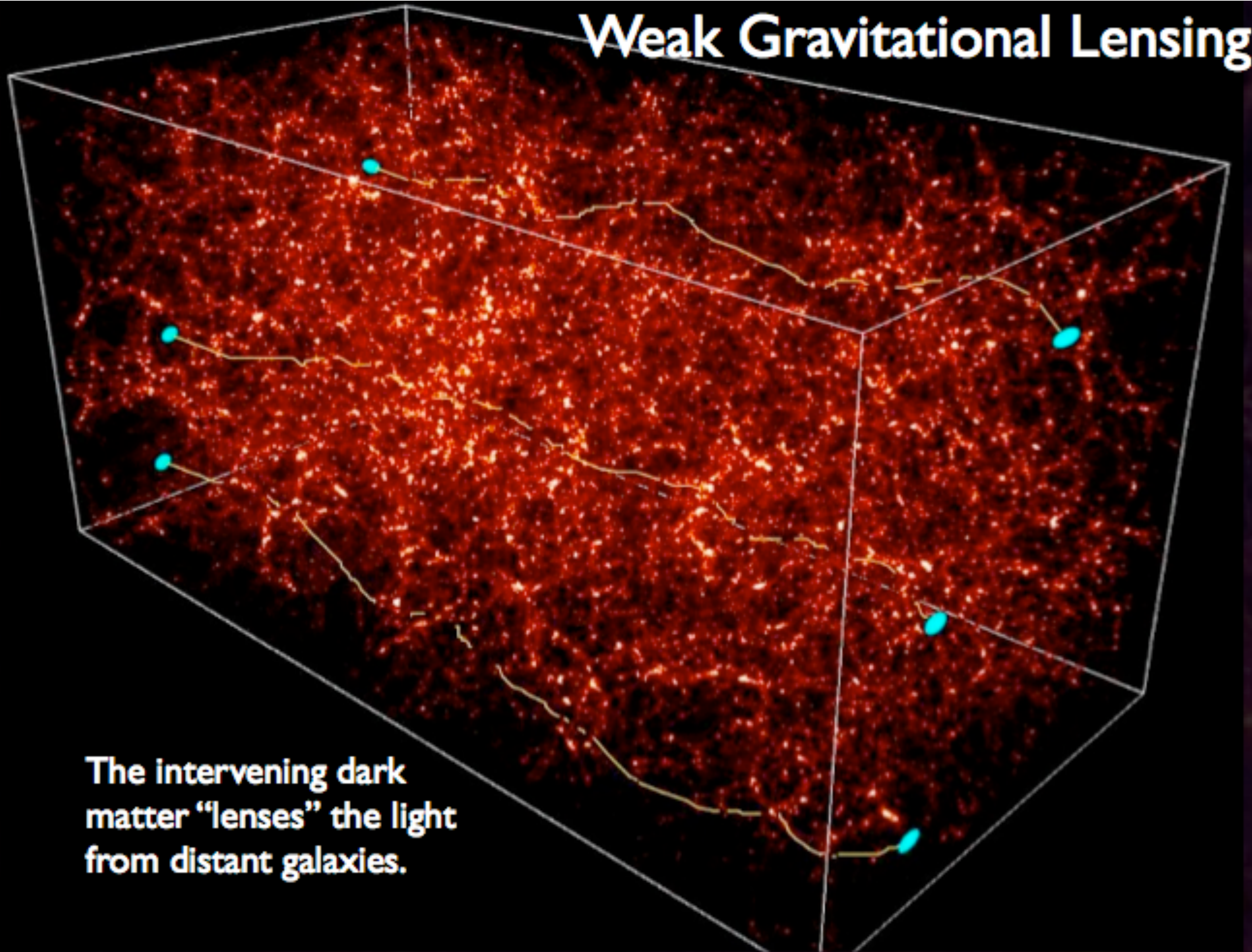






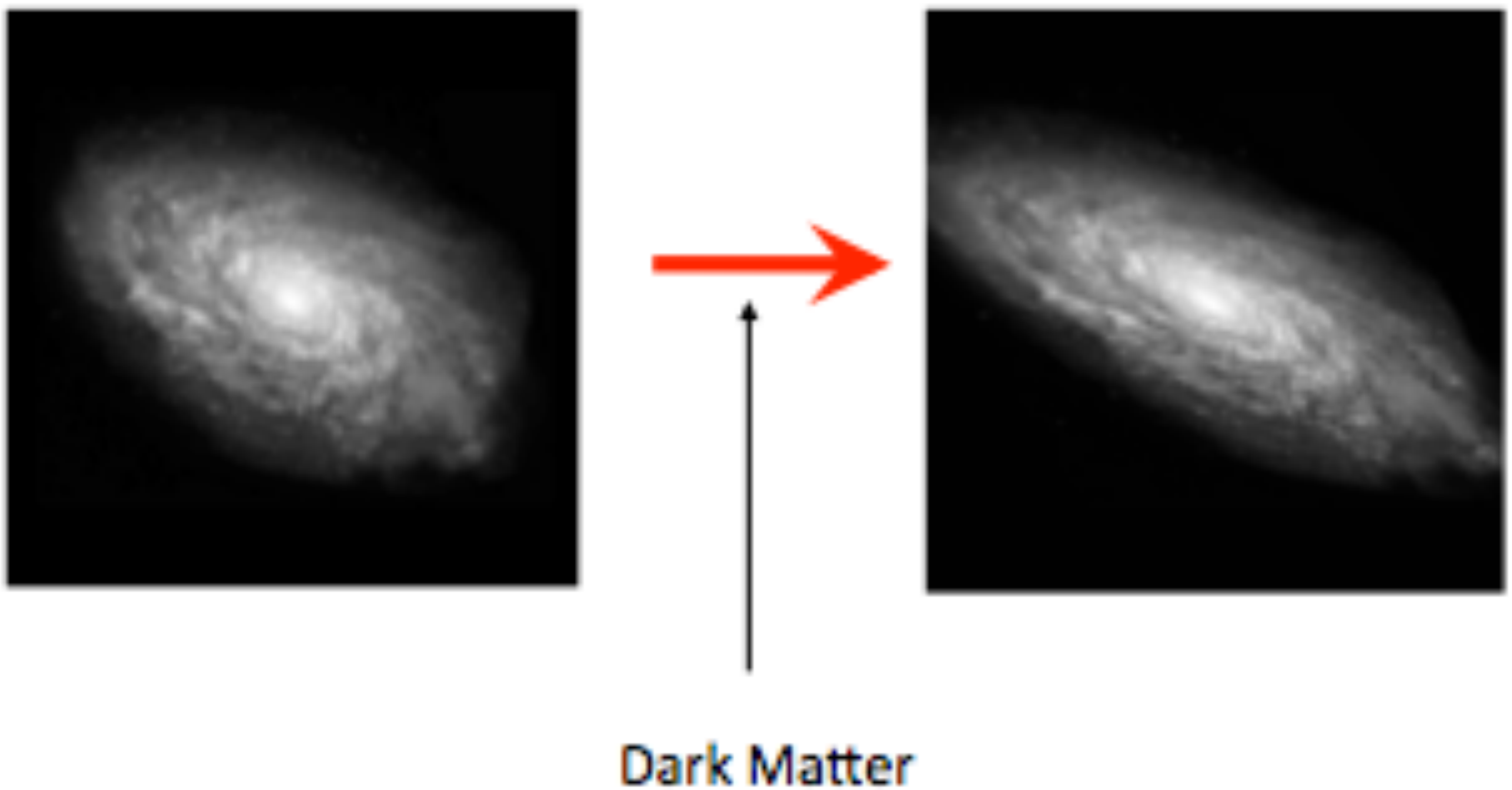
- Information on
  - Matter power spectrum
  - Angular Diameter Distance

Weak Gravitational Lensing



The intervening dark matter “lenses” the light from distant galaxies.

- The weak distortion is simply a (very small) change in ellipticity of a galaxy



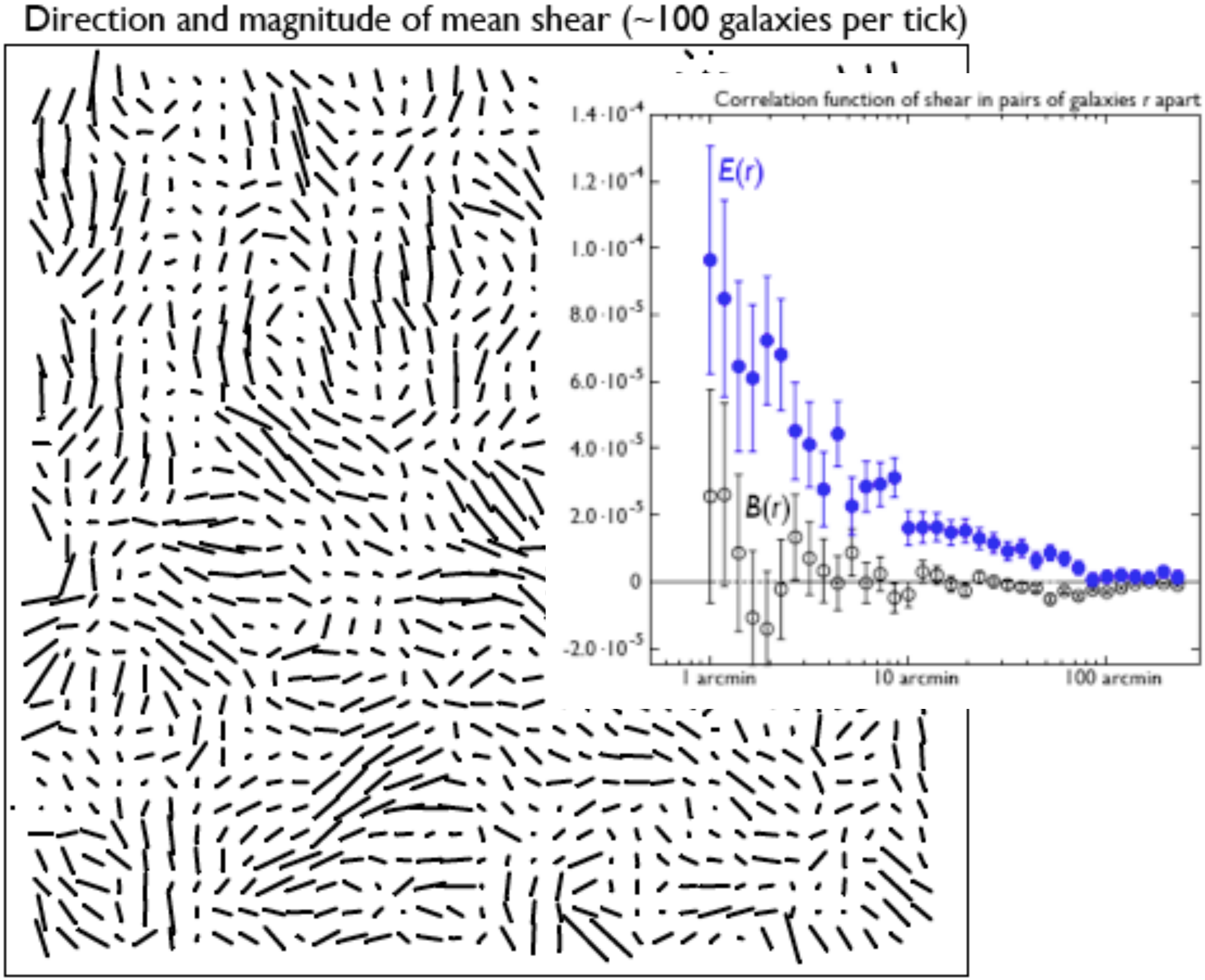
Additional ellipticity is called *__shear__*

Complex Notation

$$\gamma = \gamma_1 + i\gamma_2 = |\gamma|e^{2i\theta}$$

- Can write shear (or ellipticity) in complex form

- Shear is a *spin 2* field
  - Symmetric under rotations of 180 deg.
  - *Polarisation also an example of spin-2*
  - *Convergence is a spin-wight 0 field (symmetric under* any *rotation)*
  - *Spin 0 = scalar*
  - *Spin 1 = vector*



Mean shear is zero, but the variance contains information on cosmology

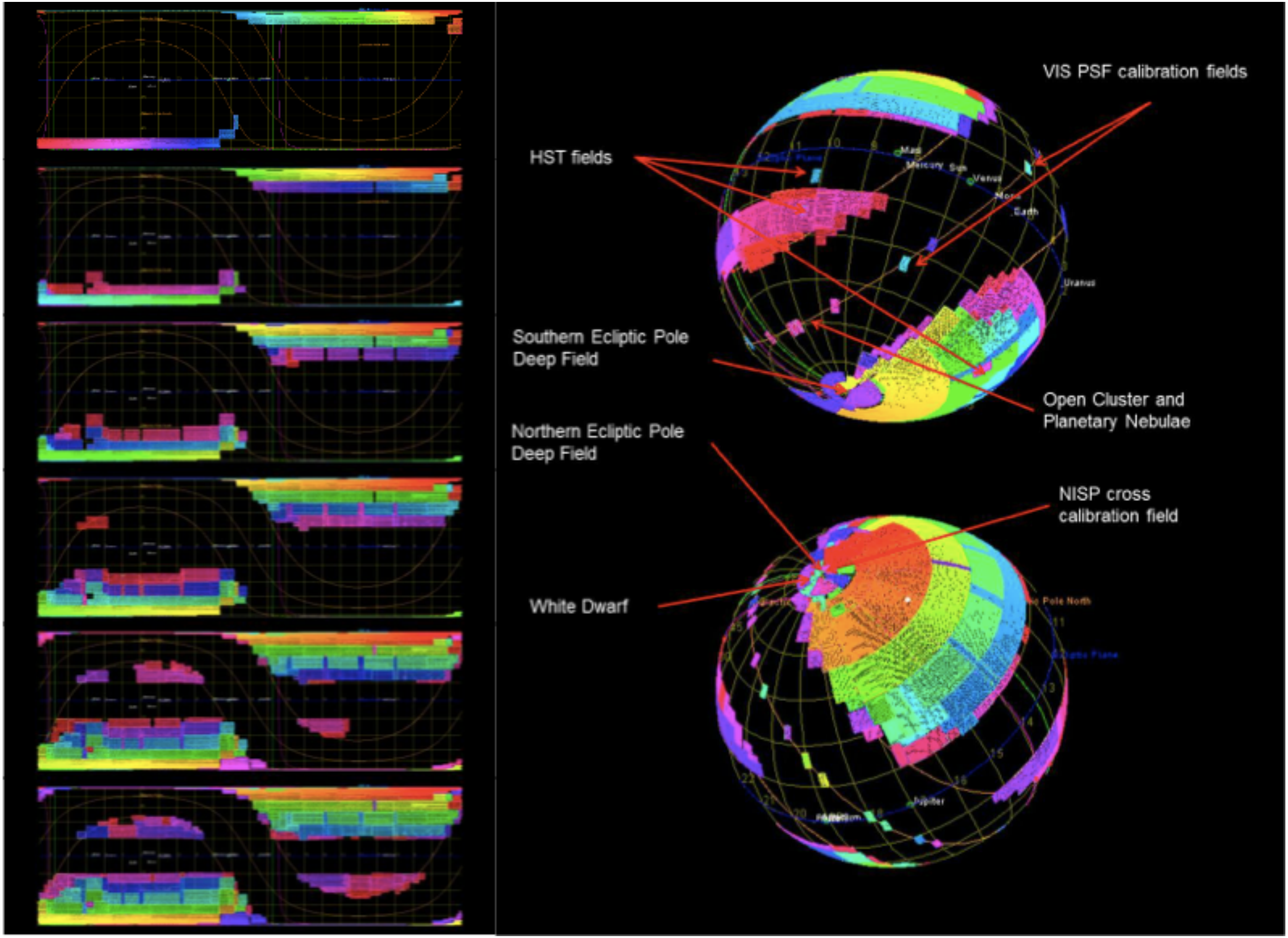
Current Weak Lensing Methods:

- ALL assume flat-sky
- ALL (except one) make shell-assumptions
- ALL are slow to run on data
- ~10 parameters estimated

Euclid/SKA/LSST (only few years away) will need:

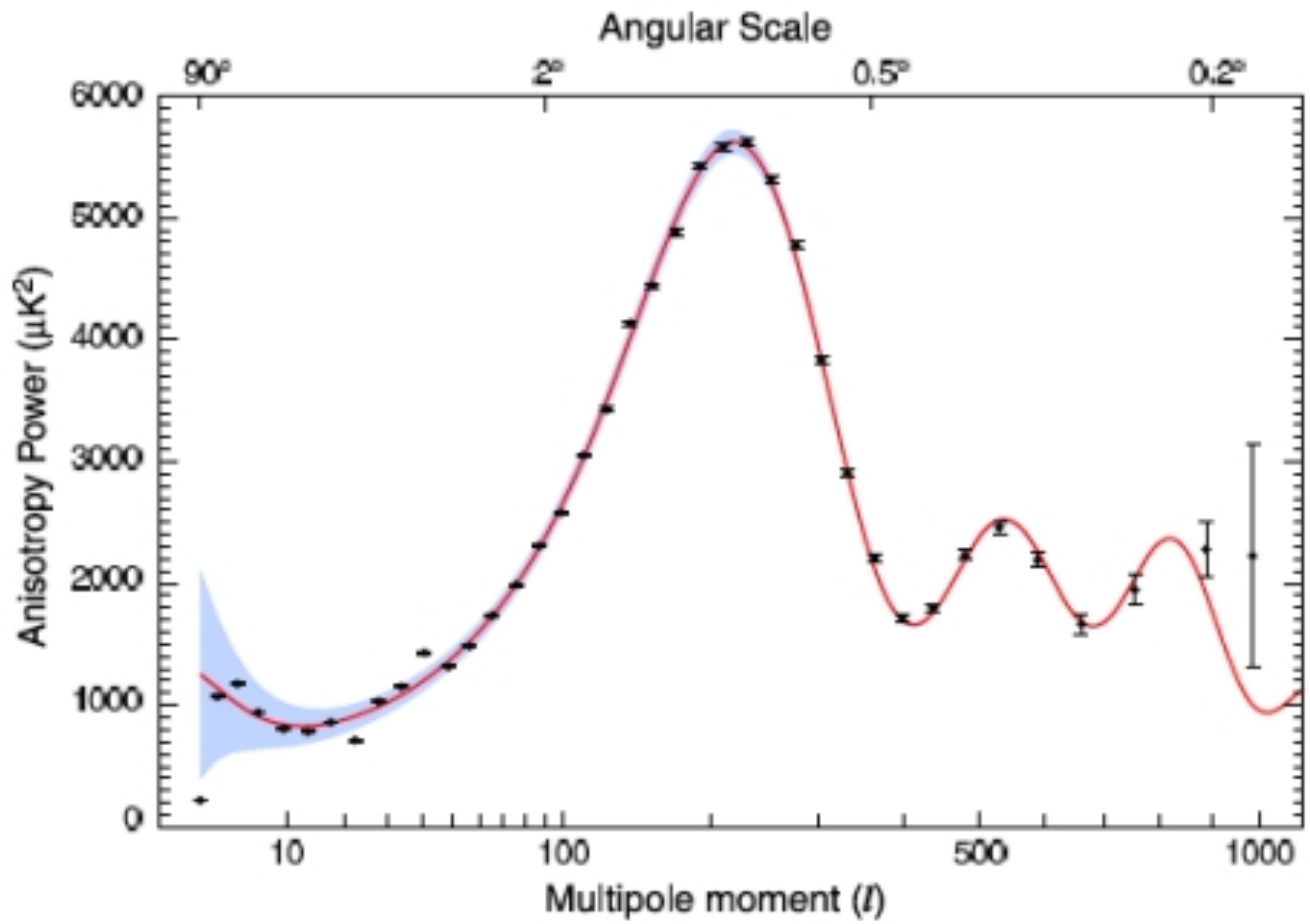
- Full spherical/ball analysis
- Fast
- ~100 parameters

-



VIS PSF calibration fields
HST fields
Southern Ecliptic Pole Deep Field
Open Cluster and Planetary Nebulae
Northern Ecliptic Pole Deep Field
NISP cross calibration field
White Dwarf

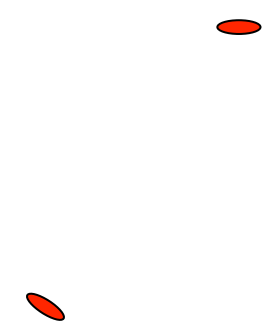
- What we want equivalent of the CMB power spectrum



Plot credit: WMAP7

But:

- CMB is a 2D field :
- Shear is a 3D field :
  - Every galaxy has a distance and a shear



- Correlation function measures the tendency for galaxies at a chosen separation to have preferred shape alignment

$$\xi(\Delta\boldsymbol{\theta}) = \int C(\boldsymbol{\ell}) \mathrm{e}^{\mathrm{i}\boldsymbol{\ell}.\Delta\boldsymbol{\theta}} \mathrm{d}^2\boldsymbol{\ell}$$

$$C(\boldsymbol{\ell}) = \frac{1}{(2\pi)^2} \int \xi(\Delta\boldsymbol{\theta}) \mathrm{e}^{-\mathrm{i}\boldsymbol{\ell}.\Delta\boldsymbol{\theta}} \mathrm{d}^2\Delta\boldsymbol{\theta}$$

Spherical Harmonics

- Normal Fourier Transform

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)\, e^{-2\pi i x \xi}\, dx,$$ for every real number ξ.

$$f(x) = \int_{-\infty}^{\infty} \hat{f}(\xi)\, e^{2\pi i x \xi}\, d\xi,$$ for every real number x.

- What we really want is the 3D power spectrum for cosmic shear
  - So need to generalise to *spherical harmonics for* spin-2 field

Spherical Harmonics

$$ {}_s f_{\ell m}(k) \equiv \sqrt{\frac{2}{\pi}} \int d^3\mathbf{r}\, {}_s f(\mathbf{r})\, k j_\ell(kr)\, {}_s Y_\ell^{m*}(\hat{\mathbf{n}}) $$

Describes general transforms on a sphere for any spin-weight quantity s

k = radial wavenumber

l and m = angular wavenumbers

Spherical Harmonics

- For flat sky approximation
  - Y's->exponentials
  - Isotropy

$$f(k, \boldsymbol{\ell}) \equiv \sqrt{\frac{2}{\pi}} \int d^3\mathbf{r} f(\mathbf{r}) k j_\ell(kr) \exp(-i\boldsymbol{\ell} \cdot \boldsymbol{\theta})$$

- Covariances of the flat sky coefficients related to the power spectrum (what we want)

$$C_\ell^{3D}(k_1,k_2) = \mathcal{A}^2 \int \mathrm{d}r_g r_g^2 n(r_g) j_\ell(k_1 r_g) \int \mathrm{d}r_h r_h^2 n(r_h) j_\ell(k_2 r_h)$$

$$\int \mathrm{d}\tilde{r}' \int \mathrm{d}\tilde{r}'' \frac{F_K(r',\tilde{r}')}{a(\tilde{r})'} \frac{F_K(r'',\tilde{r}'')}{a(\tilde{r}'')} \int \frac{\mathrm{d}k'}{k'^2} j_\ell(k'\tilde{r}') j_\ell(k'\tilde{r}'') P^{1/2}(k';\tilde{r}') P^{1/2}(k';\tilde{r}'')$$

Theory

Geometry

Large Scale Structure

$$\hat{\gamma}(k,\boldsymbol{\ell}) = \sqrt{\frac{2}{\pi}} \sum_g \gamma(\mathbf{r}) k j_\ell(k r_g^0) \exp(-i\boldsymbol{\ell}.\boldsymbol{\theta}_g)$$

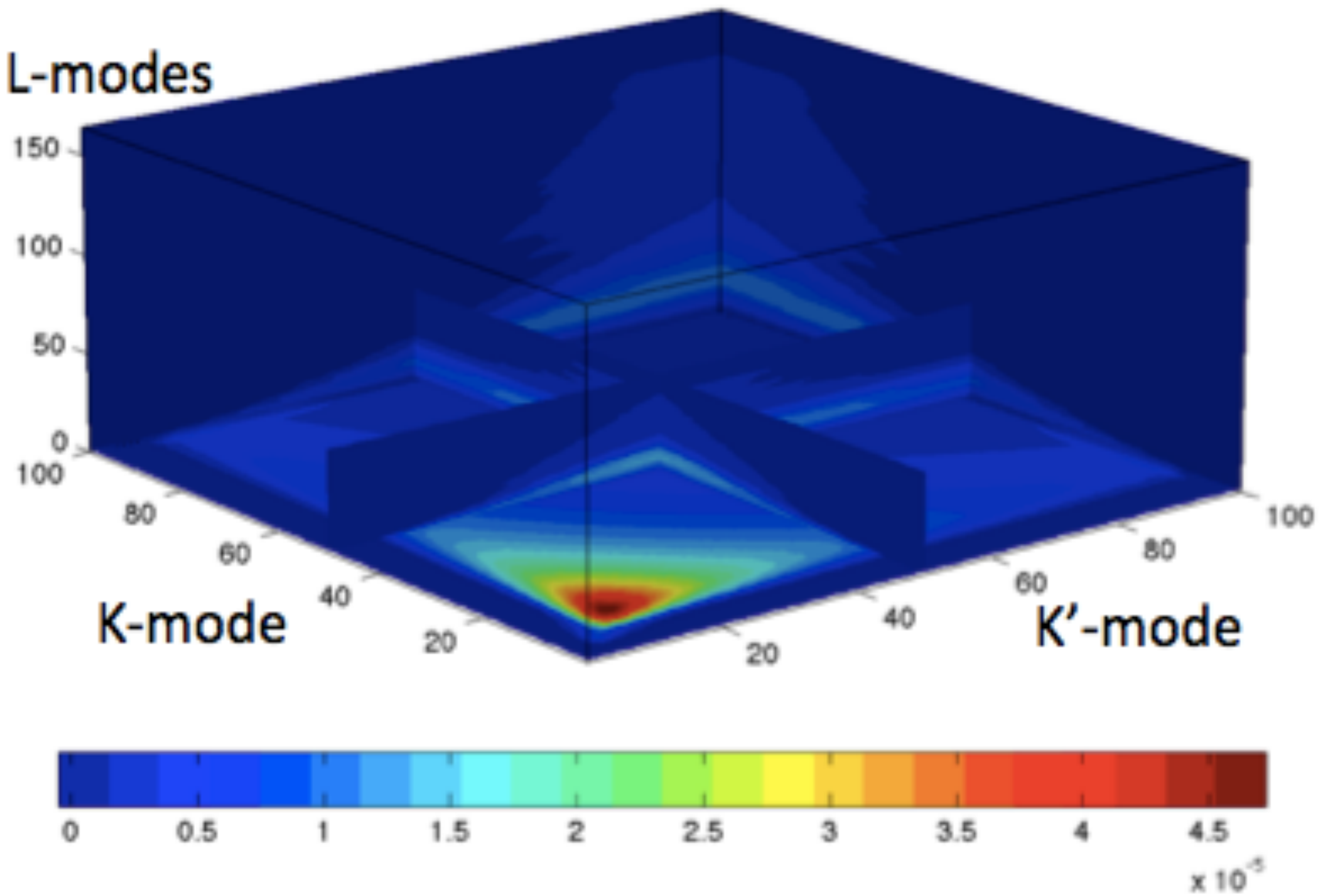
Data

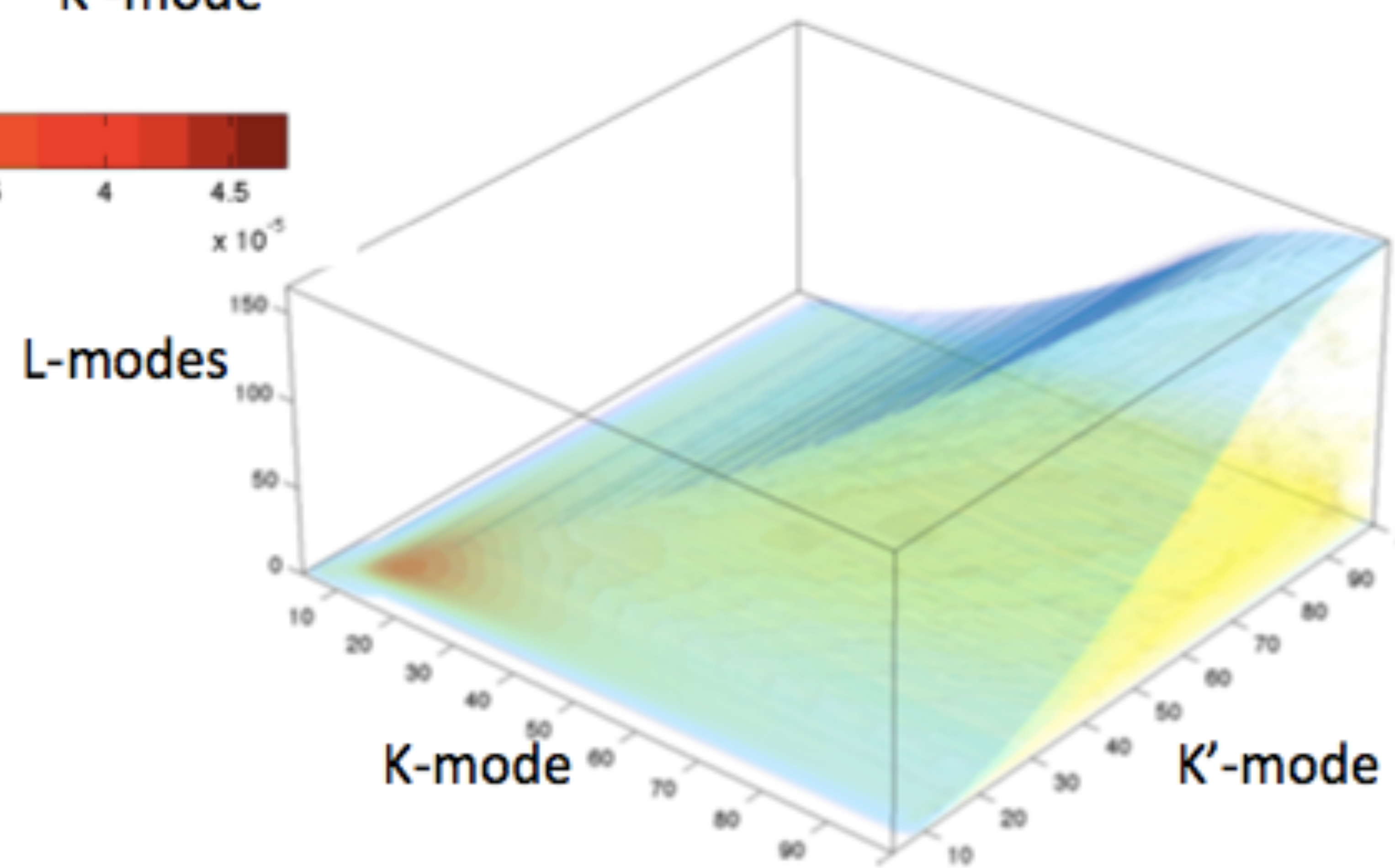
Note: can make a clean cut in scales

New code: 3Dfast to do this; send me an email

Heavens (2003)
Kitching, Heavens, Miller (2012)

– *3Dfast*


L-modes
150
100
50
0
100
80
60
40
20
K-mode
20
40
60
80
100
K'-mode
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
x 10⁻⁵


L-modes
150
100
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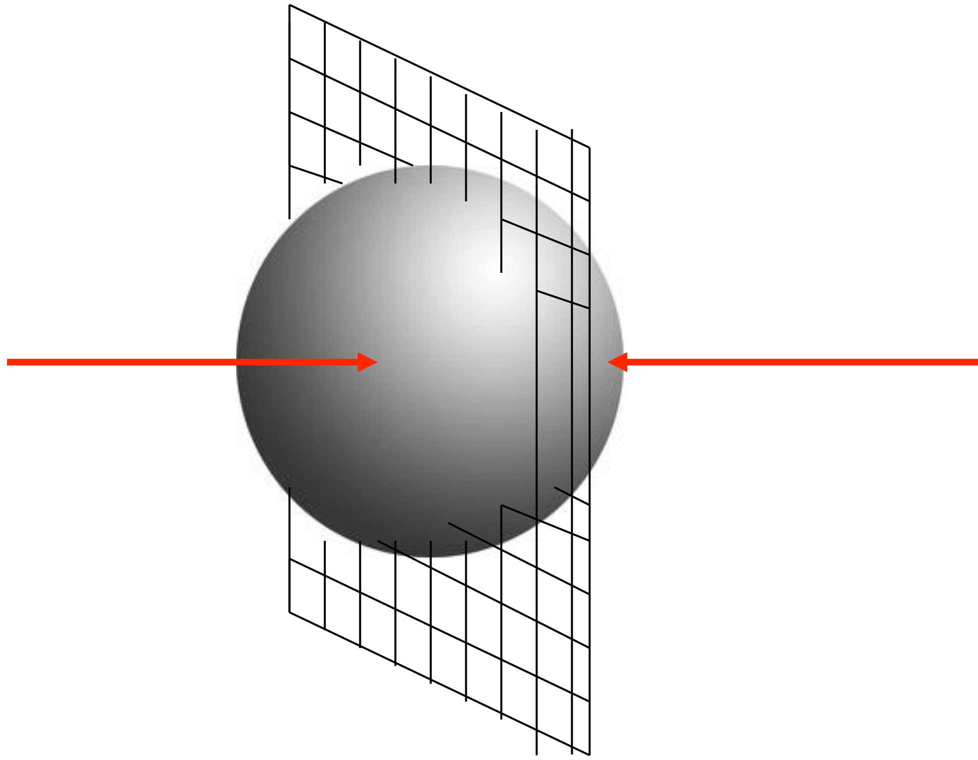
0
10
20
30
40
50
60
70
80
90
K-mode
10
20
30
40
50
60
70
80
90
K'-mode

“Tomography”/Shell Approximation

- Vast majority of community uses
  - “Tomography” in real space
  - Split into “bins” in distance/redshift

- How does it relate to the full 3D shear field?
- The “Limber Approximation”
  - (k_x,k_y,k_z) projected to (k_x,k_y)



“Tomography”

- Limber may be ok at very small scales

- Very useful Limber Approximation formula (LoVerde & Afshordi, 2010)

$$\lim_{\ell \to \infty} j_\ell(kr) \to \sqrt{\frac{\pi}{2\left(\ell + \frac{1}{2}\right)}} \delta^D \left(kr - \left[\ell + \frac{1}{2}\right]\right)$$

$$C_\ell^{3D}(k_1,k_2) = \mathcal{A}^2 \int \mathrm{d}r_g r_g^2 n(r_g) j_\ell(k_1 r_g) \int \mathrm{d}r_h r_h^2 n(r_h) j_\ell(k_2 r_h) .$$
$$\int \mathrm{d}\tilde{r}' \int \mathrm{d}\tilde{r}'' \frac{F_K(r',\tilde{r}')}{a(\tilde{r})'} \frac{F_K(r'',\tilde{r}'')}{a(\tilde{r}'')} \int \frac{\mathrm{d}k'}{k'^2} j_\ell(k'\tilde{r}') j_\ell(k'\tilde{r}'') P^{1/2}(k';\tilde{r}') P^{1/2}(k';\tilde{r}'')$$

$$C_\ell^{3D,\mathrm{Limber}}(k_1,k_2) = \frac{9\Omega_m^2 H^4}{4c^2} \int \mathrm{d}r \frac{P(\ell/r;r)}{a^2(r)} \frac{\mathcal{W}(r_1,r)\mathcal{W}(r_2,r)}{r^2}$$

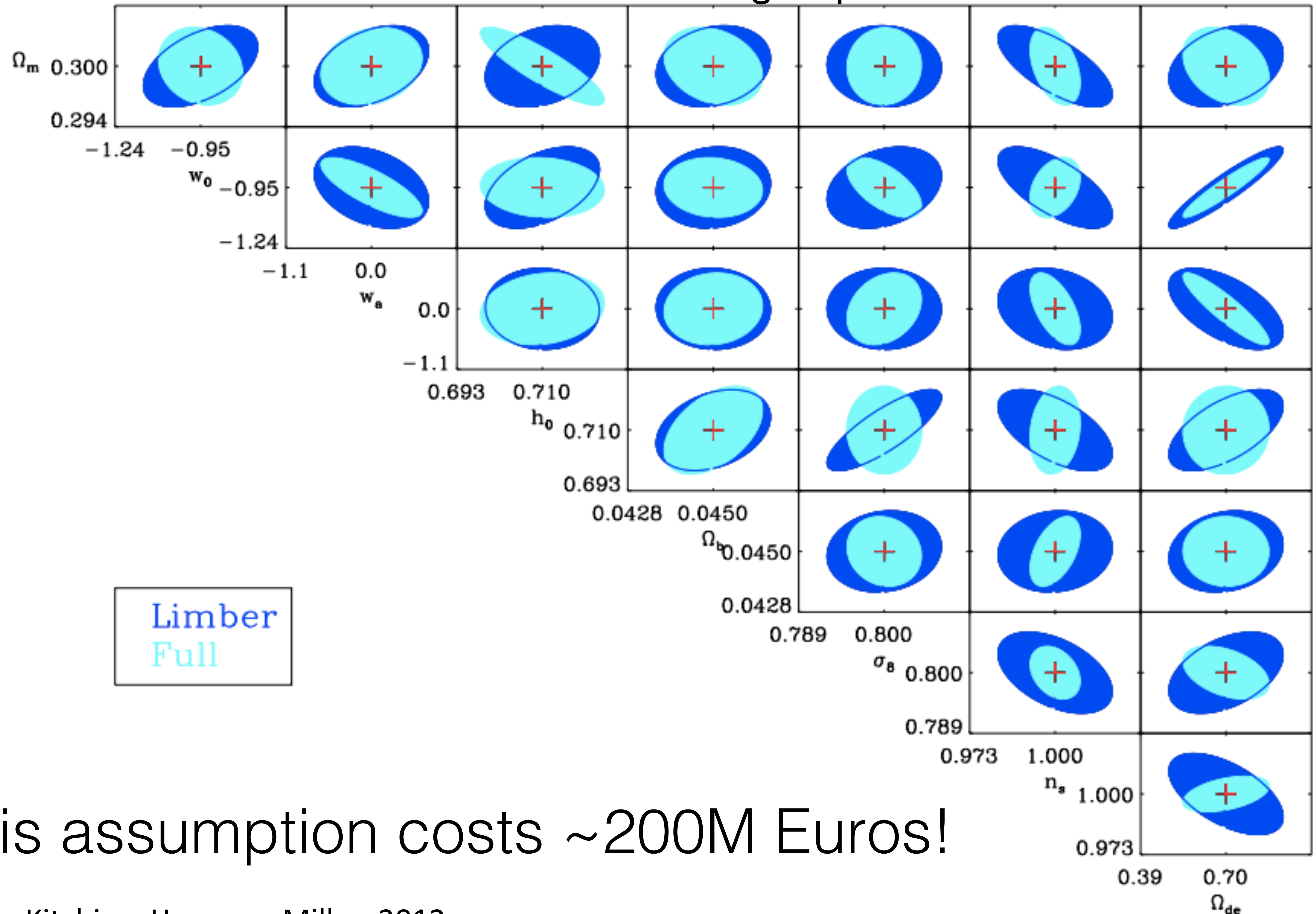
Note that k-mode is now linked to l-mode
k=l/r

The "Tomographic" Approximation

$$C_l(z_l, z_2) = \frac{9\Omega_m^2 H^4}{4c^2} \int \mathrm{d}r \frac{P(\ell/r; r)}{a^2(r)} \frac{\mathcal{W}(r_1, r)\mathcal{W}(r_2, r)}{r^2}$$

- An approximation to the 3D power
- Do we even want to do "tomography"?! Why?
- Approximations
  - Limber Approximation (lossy)
    - replaces Bessels with Dirac delta functions
  - Transform to Real space (benign)
  - Discretisation in redshift space (lossy)

Predicted Fisher Matrix Errors on some cosmological parameters



This assumption costs ~200M Euros!

Kitching, Heavens, Miller, 2012

- The aggregated effect of marginal gains

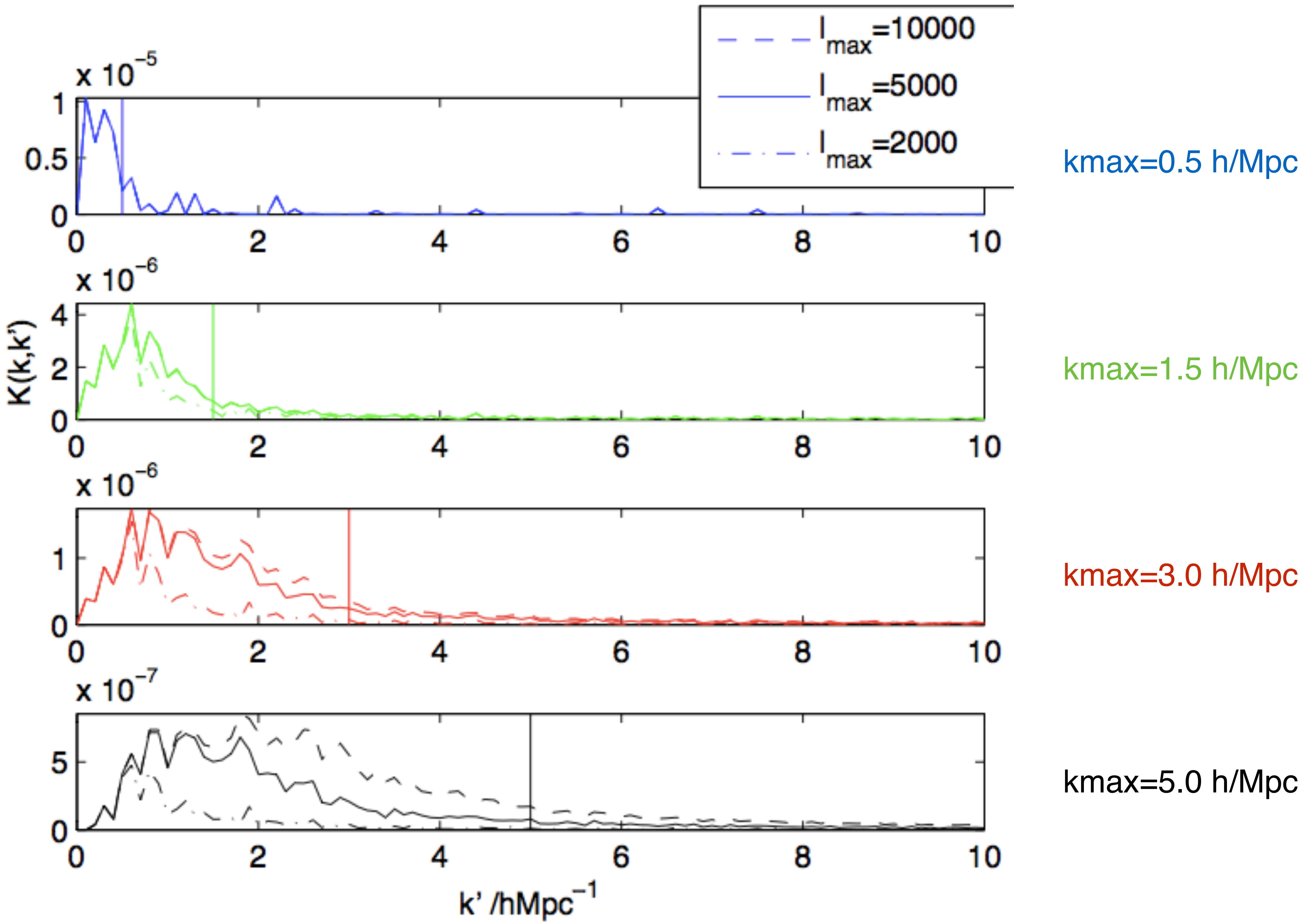
What is the Kernel?

$$C_\ell^{3D}(k_1,k_2) = \mathcal{A}^2 \int \mathrm{d}r_g r_g^2 n(r_g) j_\ell(k_1 r_g) \int \mathrm{d}r_h r_h^2 n(r_h) j_\ell(k_2 r_h) \int \mathrm{d}r' \bar{p}(r'|r_g) \int \mathrm{d}r'' \bar{p}(r''|r_h) \int \mathrm{d}\tilde{r}' \int \mathrm{d}\tilde{r}'' \frac{F_K(r',\tilde{r}')}{a(\tilde{r})'} \frac{F_K(r'',\tilde{r}'')}{a(\tilde{r}'')} \int \frac{\mathrm{d}k'}{k'^2} j_\ell(k'\tilde{r}') j_\ell(k'\tilde{r}'') P^{1/2}(k';\tilde{r}') P^{1/2}(k';\tilde{r}'')$$

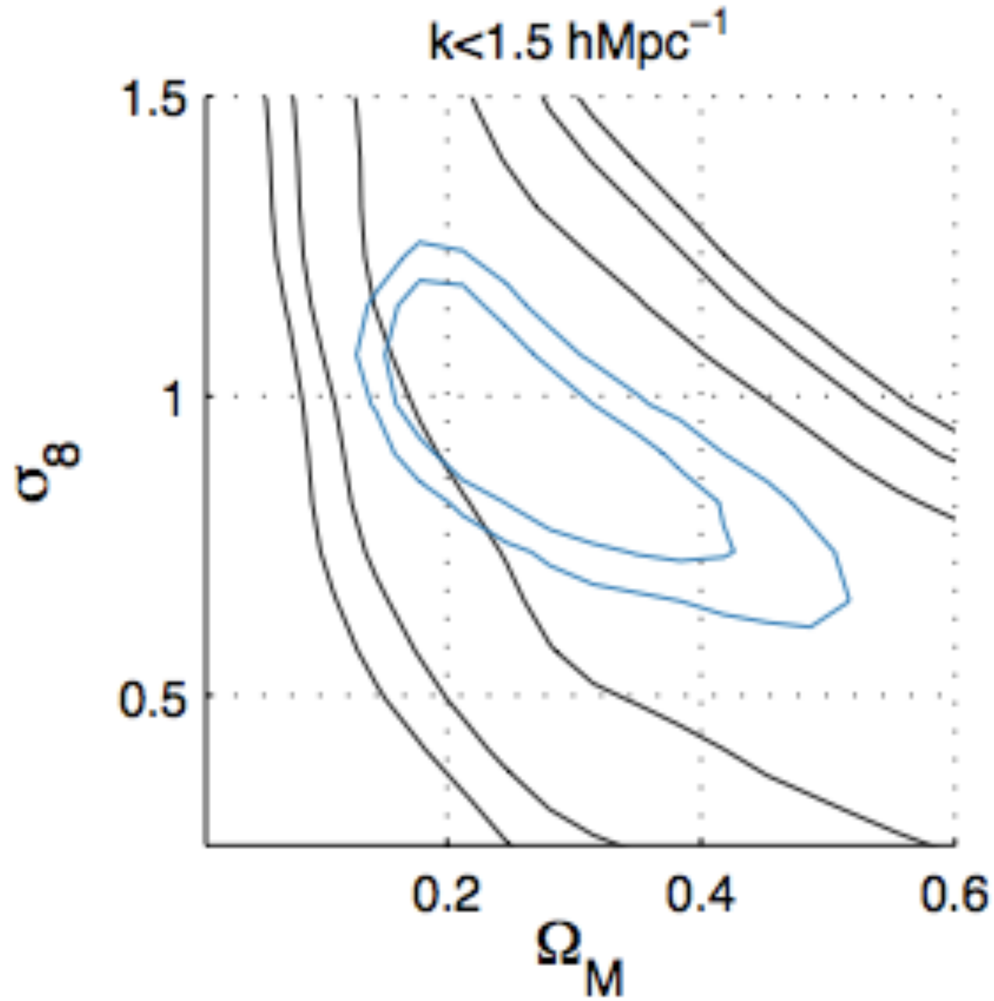
$$C_\ell^S(k,k) = \int P(k';z) K(k',k) \mathrm{d}k'$$

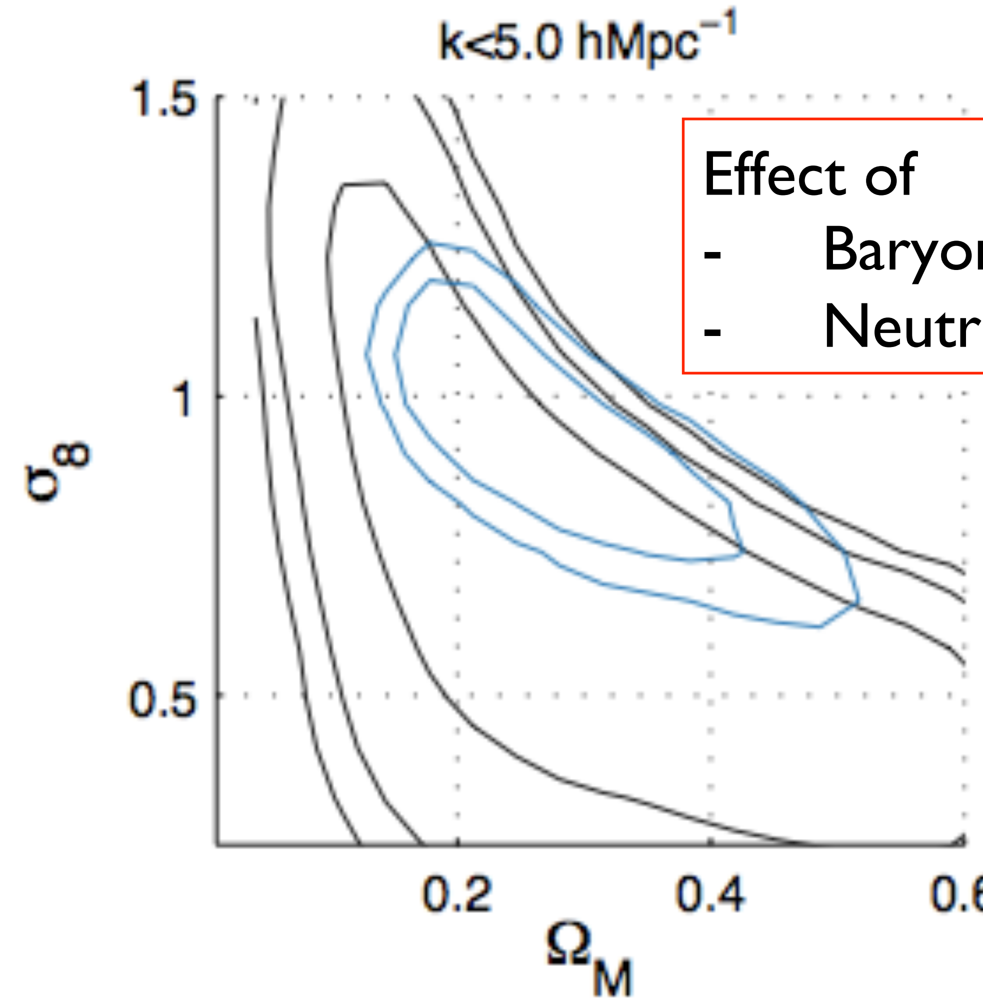
Kernel

$$C_\ell^S(k,k) = \int P(k';z)K(k',k)\mathrm{d}k'$$

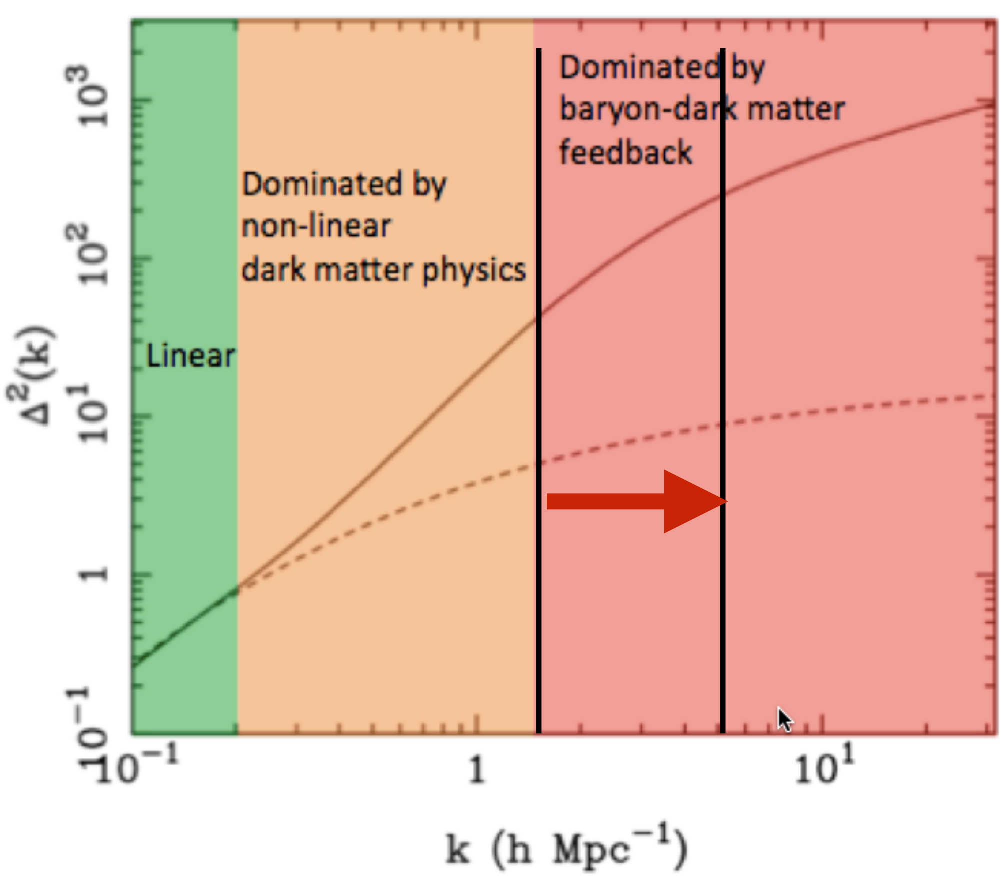


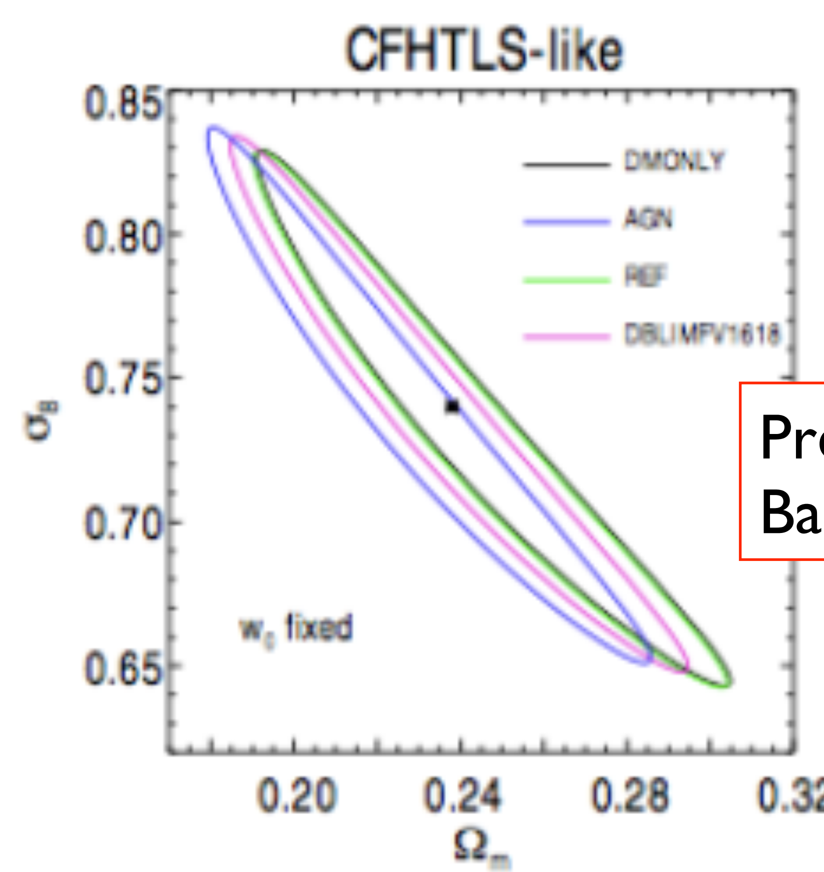
Compared to: $\xi_\pm(\vartheta) = \frac{1}{2\pi}\int_0^\infty \mathrm{d}\ell\,\ell\,[P_\mathrm{E}(\ell) \pm P_\mathrm{B}(\ell)]\,\mathrm{J}_{0,4}(\ell\vartheta)$





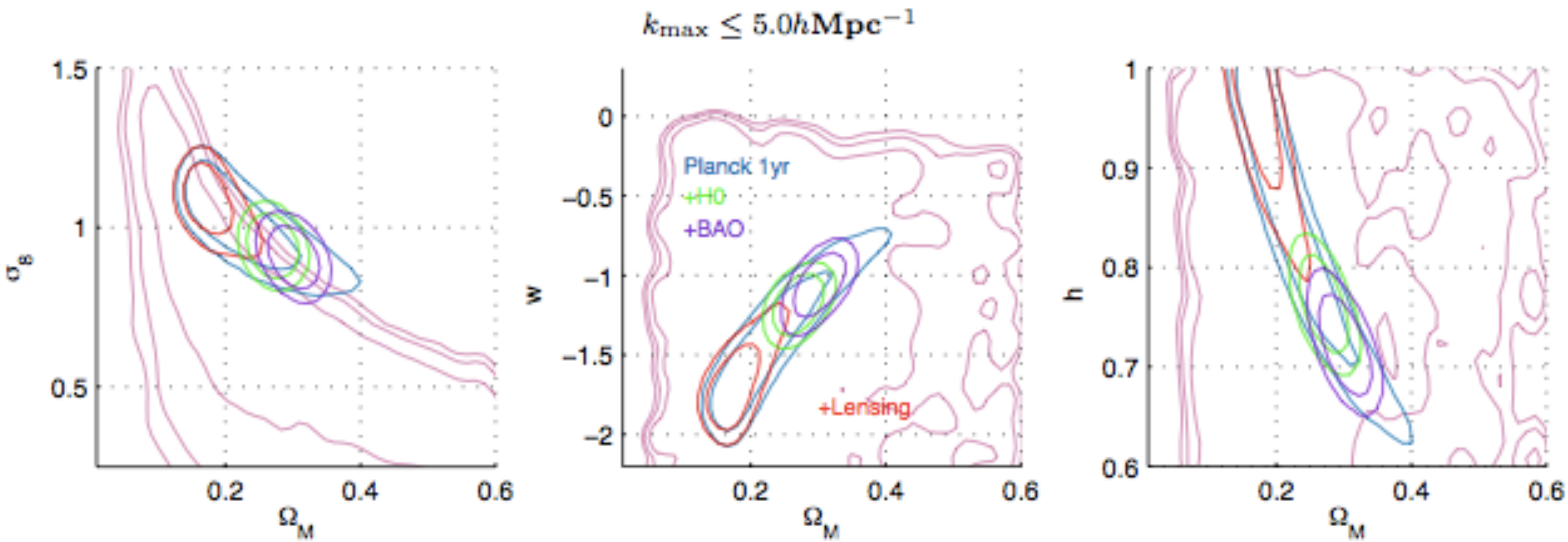
Effect of
- Baryonic Feedback
- Neutrinos





Prediction for
Baryonic feedback

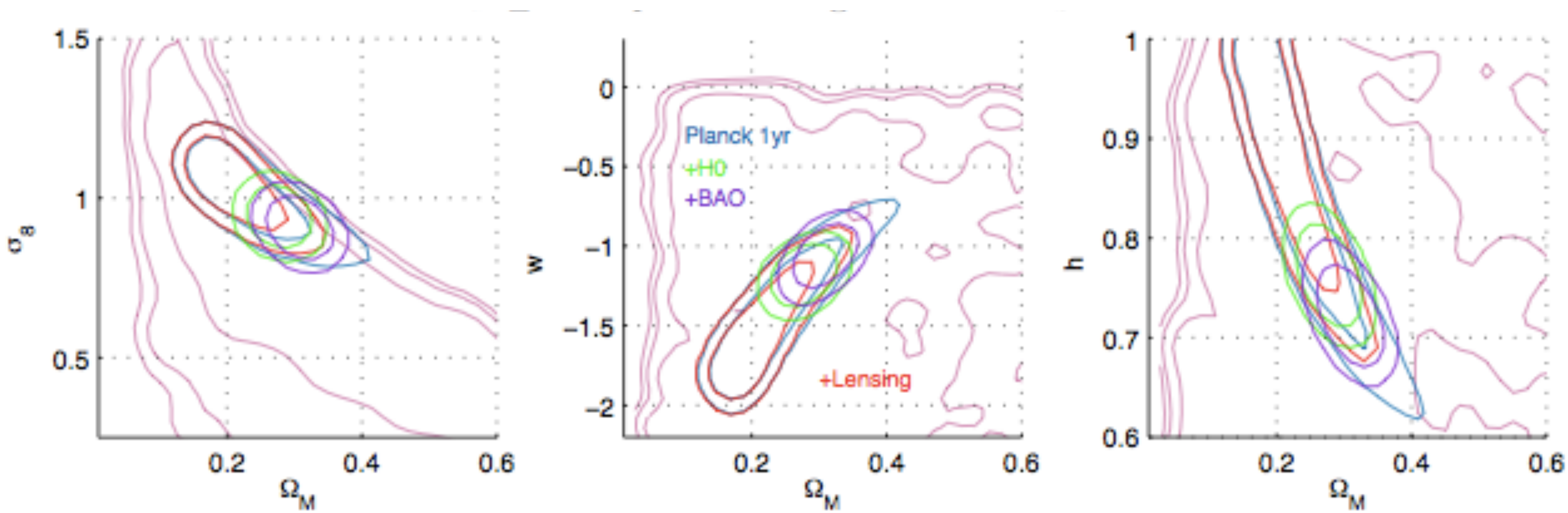
Tension with Planck?



Tension with Planck?

No.

Including a functional ansatz to account for AGN feedback from Semboloni et al. (2013) :



- 3D Spherical Bessel analysis of weak lensing data
  - Computationally expensive
  - Includes information without need for binning
  - Can exclude particular scales that may be dominated by poorly understood systematic effects
- Relaxing assumptions
  - Flat Sky – to Spherical !
  - Spherical Bessel expansion
    - What is the optimal weight function/basis
- Generalizations
  - Beyond 2-point statistics
  - How to incorporate compressed sensing & wavelets
    - What is the optimal sampling in l and k?